Interdependence, complementarity, and ruggedness of performance landscapes

Hazhir Rahmandad  
MIT  
Sloan School of Management  
hazhir@mit.edu

Abstract

Performance landscape -- the mapping between firm choices and performance outcomes? is a construct central to understanding organizational adaptation and heterogeneity. Yet prior formalizations split between unimodal production functions and rugged landscapes with many local peaks. Capturing interaction effects among organizational choices I introduce the PN landscape which applies to both continuous and discrete choices and includes the well-known NK architecture as a special case. Using this formalization I explore the robustness of intuitions accompanying NK theorizing finding that: (1) The number of local peaks may grow much more slowly with number of choices than previously formalized; and (2) Complementarity significantly reduces the number of peaks. In fact under maximum complementarity (i.e. supermodularity) one rarely observes more than two local peaks, and that number declines with N. Moreover, modeling the evolution of landscapes in light of innovation offers a bridge between the divergent views of performance landscapes. Over time performance enhancing (innovative) mutations increase complementarity and reduce the number of local peaks, potentially to a single one. Results highlight the relevance of landscapes with a few local peaks where management’s role is more salient than either extreme, and provide an explanation for the emergence of complementarity in organizational activity systems.
Interdependence, complementarity, and ruggedness of performance landscapes

Hazhir Rahmandad, MIT

ABSTRACT

Performance landscape -- the mapping between firm choices and performance outcomes— is a construct central to understanding organizational adaptation and heterogeneity. Yet prior formalizations split between unimodal production functions and rugged landscapes with many local peaks. Capturing interaction effects among organizational choices I introduce the PN landscape which applies to both continuous and discrete choices and includes the well-known NK architecture as a special case. Using this formalization I explore the robustness of intuitions accompanying NK theorizing finding that: (1) The number of local peaks may grow much more slowly with number of choices than previously formalized; and (2) Complementarity significantly reduces the number of peaks. In fact under maximum complementarity (i.e. supermodularity) one rarely observes more than two local peaks, and that number declines with N. Moreover, modeling the evolution of landscapes in light of innovation offers a bridge between the divergent views of performance landscapes. Over time performance enhancing (innovative) mutations increase complementarity and reduce the number of local peaks, potentially to a single one. Results highlight the relevance of landscapes with a few local peaks where management’s role is more salient than either extreme, and provide an explanation for the emergence of complementarity in organizational activity systems.

INTRODUCTION

Performance landscape describes the mapping between a firm’s choices and its expected performance. Explicitly, or implicitly, the shape of performance landscape is central to our understanding of organizations. For example, a rugged landscape (Levinthal 1997) underscores local adaptation to various local performance peaks and emergence of heterogeneity in organizational forms and outcomes. In contrast, a landscape with a single peak indicates a unique best strategy that rational managers find
analytically and more behavioral agents can find through search heuristics. More generally, search processes and outcomes are tightly connected with the shape of performance landscapes (Fleming and Sorenson 2001, Sorenson et al. 2006). Radical organizational change may entail moving across different local performance peaks creating significant risks and temporal tradeoffs (Pil and MacDuffie 1996). Performance landscapes may change as a result of technological innovations (Henderson and Clark 1990), leaving incumbent firms exposed to competency traps (Leonard-Barton 1992, Levitt and March 1988) and flawed mental models (Benner and Tripsas 2012, Tripsas and Gavetti 2000). Market level competitive dynamics and selection processes are also a function of underlying performance landscape (Lenox et al. 2006). In short, we can’t claim to understand, or improve, many organizational problems if we can’t formalize, measure, and analyze performance landscapes.

However, despite its key role in understanding organizations, there is little agreement about the shape of performance landscapes. Two very different perspectives have emerged. The literature influenced by economics uses production functions to represent performance landscapes. Most common production functions, seeking analytical tractability, focus on tradeoffs in continuous choice spaces and have a (single peaked) concave topology (Mas-Colell et al. 1995, Shephard 1970). This view is reinforced by empirical studies that estimate these production functions (Murillo-Zamorano 2004) and explains differences among firms in terms of responses to different target markets and environments, i.e. differences in the underlying performance landscapes (Osterman 2018, Porter 1998) rather than multiple local peaks on a single landscape.

In contrast, an important alternative has conceptualized firms as systems of interdependent activities and choices (Fleming 2001, Levinthal 1997, Siggelkow 2011), arguing that the shape of performance landscape should emerge from those interactions. Building on the observed complementarities among elements of firm strategy (Milgrom and Roberts 1995), the NK performance landscape (Levinthal 1997) has emerged as the leading formalization of how those interactions shape the organizational performance landscapes (Bauman et al. 2019). In this landscape the mapping between strategies and performance outcomes has many local peaks recasting strategy and performance
heterogeneity as the outcome of a complex search process. The resulting complexity penalizes efforts to find the global maximum and favors local adaptation; local peaks then become attractors for diverse organizational configurations. The NK landscape not only has provided a formal architecture to explain heterogeneity in organizational forms and outcomes, but also has become a fruitful vehicle for exploring various organizational phenomena from learning mode (e.g., Levinthal 1997, Rivkin 2000, 2001) to distributed search, modularity and problem decomposition (e.g., Ethiraj and Levinthal 2004, Marengo et al. 2000, Siggelkow and Rivkin 2009), cognitive maps (e.g., Csaszar and Levinthal 2016, Gavetti and Levinthal 2000), temporal dynamics (e.g., Baumann and Siggelkow 2013, Levinthal and Posen 2007), and positioning and competition (e.g., Adner et al. 2014, Gavetti et al. 2017, Lenox et al. 2006), among others (see (Bauman et al. 2019) for a recent review). Beyond intricate modeling insights, the NK landscape has been a powerful metaphor promoting a few broader intuitions about organizational performance landscapes: that strategy spaces are better represented as discrete choices; that the number of local peaks grows very rapidly with the number of choices (N) and their interdependence (K); and that complementarity is a major source of ruggedness on landscapes.

Intuitions embedded in the NK formalization come in sharp contrast against the properties of performance landscapes formalized by economists. That leaves students of organizations with a stark choice between performance landscapes that are polar opposites. There are few bridges between the two and little guidance on what features of organizational choice systems should inform the choice of appropriate formalization. For example, we know rather little about the robustness and generalizability of the influential intuitions accompanying the NK landscape. Do they apply more broadly to alternative formalizations of interactions among elements of firm strategy? Are they supported by any fundamental organizational mechanism regulating the formation and evolution of performance landscapes? Could one bridge between the vastly different conceptions of performance landscapes embedded in production functions and NK formalism?

In this paper I take a first step towards examining the robustness and boundary conditions of intuitions embedded in the NK formulation, with an eye on building a bridge between the opposing
conceptions of performance landscapes. I start by formulating a performance landscape capturing all possible interactions among organizational choices that not only encompasses NK as a special case, but also can apply to continuous choices. Using this generalization I revisit some of the intuitions that have accompanied the NK research, finding that: 1) This expansion of landscape to continuous spaces closely follows NK in that it does not introduce any internal local peaks. Nevertheless, the expected number of local peaks in this alternative formalization of interdependencies grows much more slowly than in the canonical NK. 2) Size of basins of attraction become more unequal as a result of both the new conception of interdependence, and the continuous strategy space, concentrating outcomes of local adaptation among fewer peaks. 3) Under strong complementarity (i.e. supermodularity) one rarely finds more than two local peaks regardless of N and K. More generally complementarity significantly reduces the number of peaks. 4) When performance landscapes are allowed to incorporate performance enhancing innovations the complementarity among choices increases and the number of local peaks goes down sharply, shifting the landscape towards a unimodal (single-peaked) one over the long-run. These results offer a bridge between the diverging views of performance landscapes, highlights the relevance of landscapes with modest ruggedness where the role of management would be more pronounced, and provides a process view for studying the emergence of landscapes and the complementarity observed in activity systems.

**Complementarity and Ruggedness of Performance Landscapes**

To understand organizations we need to measure, formalize, and analyze how different organizational choice combinations lead to performance outcomes of interest. However, it is easier to theoretically define the concept of performance landscape than to formalize it based on empirical observations. It is often complex to reliably estimate the impact of any single choice on organizational performance even in a narrow range of choices empirically observed. It is a much harder task to capture the impact of all important choices and their interactions in the full strategy space (i.e. the space encompassing feasible range of all organizational choices). Thus researchers have taken two distinct approaches to formalize a performance landscape. The dominant approach in economics reduces the dimensionality of strategy space by focusing on a handful of aggregate choices (e.g. labor and capital), assumes a smooth and often
concave functional form that is analytically tractable (Mas-Colell et al. 1995, Shephard 1970), and uses empirical data to estimate the few parameters defining this function (Murillo-Zamorano 2004). In contrast, a growing number of organizational scholars have embraced the complexity implied by viewing firms as systems of interdependent choices (Fleming 2001, Porter 1996, Siggelkow 2011), and have formulated performance landscapes that reflect this complexity. Chief among these is the NK landscape. It originated in evolutionary biology (Kaufmann 1993), was introduced to organizational scholars by Levinthal (1997), and has since become a productive vehicle for analyzing diverse organizational phenomena, as well as a powerful metaphor informing general intuitions about the shape of, and adaptation on, organizational performance landscapes.

In the canonical NK landscape (Levinthal 1997, Rivkin 2000) each organizational configuration (i.e. a point on the strategy space) is formalized as $N$ binary choices (e.g. 0 or 1), creating a strategy space $2^N$-points large. The performance of each configuration is the average of contributions from the $N$ different choices. Those contributions ($c_i$) are randomly drawn and, besides the focal choice, depend on $K$ other choices (thus the $K$, in NK) based on random or other interdependency matrices (Rivkin and Siggelkow 2007). Formally, the payoff $\pi_{NK}(s)$ for the choice vector $s$ (vectors are shown in bold) is defined in equation 1, where $C$ values are drawn from a uniform random distribution for every unique combination of $s_i$ and the $K$ other choices it depends on ($s_i; s_{i1}, ..., s_{iK}$):

$$\pi_{NK}(s) = \frac{\sum_{i=1}^{N} c_i(s_i; s_{i1}, ..., s_{iK})}{N}$$  \hspace{1cm} (1)

The notion of a peak on a landscape is key to understanding the main results in this literature. Specifically, a local peak is a point that outperforms all its immediate neighbors. In the NK’s discrete strategy space an immediate neighbor of point $s$ is one with $N-1$ choices identical to $s$ and one choice that is different (thus each point has $N$ neighbors). Local moves, analogous to incremental organizational adaptation, are defined as those moves taking an organizational configuration to a neighboring point. A global peak is one that has a higher payoff than any other point on the landscape; and a peak’s basin of attraction is the region of landscape from which local hill-climbing (i.e. local moves that improve
performance) will eventuate at that peak. In the NK formulation, when $K=0$ the contribution of each choice is independent of other choices and thus can take only two values, one for $s_i=0$ and another for $s_i=1$. The payoff function then becomes linear with respect to $s$, and would have a single peak on this strategy space that is also the global peak. On the other extreme, when $K=N-1$, the contribution of each choice to the payoff function is different and independent across every two points, creating completely independent payoffs across all points on the strategy space. Any point can then be a local peak with probability $1/N$, creating a landscape with $2^N/(N+1)$ peaks in expectation. Thus the number of local peaks grows exponentially with $N$ (and $K$); while expected number of peaks is fixed for $K=0$ and $K=N-1$, pattern of interdependence among choices impacts that number for $K$s between the two limits (Rivkin and Siggelkow 2007). Moreover, with higher $K$ lack of correlation across neighboring points’ payoffs minimizes information gained about the other regions of the landscape after visiting each configuration, requiring exhaustive search, and creating an intractable optimization problem to find the global peak when $N$ and $K$ are large (Rivkin 2000). Organizational inertia (Hannan and Freeman 1984), coordination challenges (Milgrom and Roberts 1995, Pil and MacDuffie 1996), and narrow cognitive maps (Gavetti 2012) make it likely that most changes in organizational configurations and strategies are best represented as local moves. Major reorientations (Romanelli and Tushman 1994), captured as *Long Jumps* to distant regions of strategy space (Levinthal 1997), are possible, but costly and with uncertain rewards and thus would be limited in their frequency (Hannan and Freeman 1984). Therefore organizations, unable to “solve” the strategic choice problem, are largely focused on local adaptation and likely to end up in local peaks, the number of which informs the heterogeneity in organizational forms and outcomes.

The resulting platform not only has offered a coherent explanation for understanding heterogeneity in organizational forms and outcomes, but also has underpinned a large portion of simulation-based research in organization theory. In fact, the number of articles using the NK landscape to study organizational phenomena has been on a steadily rise since late 1990s and the cumulative count now exceeds 70 papers in prominent journals (Bauman et al. 2019). A review of this diverse literature is
beyond the scope of the current paper but is available at different levels of detail and topical focus (Bauman et al. 2019, Ganco and Hoetker 2009, Porter and Siggelkow 2008, Puranam et al. 2015).

The impact of this platform on organizational scholarship goes beyond the intricate mechanisms documented in this expanding literature. Most students of organizations are less interested in the details of simulation models than in the broader intuitions accompanying models. For example, the economic production functions have built intuitions about continuous and low-dimensional strategy spaces with unimodal performance landscapes that permeate much economic thinking about organizations. In contrast the NK platform has provided intuitions about the importance of discrete; high-dimensional strategy spaces; how complementarity justifies rugged (multi-peaked) landscapes; and the large number of peaks growing with interdependency among organizational choices. The gap between these competing views of organizations is large and few have attempted to bring them closer. For example, the choice of concave, unimodal, production functions downplays the role of search processes because not only rational firms, but also firms following simple search heuristics, should be able to find the single optimal strategy (Levinthal 2011). Such view also finds heterogeneity among organizations a surprising phenomenon (Syverson 2011). Besides the divergence among existing views on the number of local peaks, two other intuitions of NK landscapes can benefit from a closer look.

One key feature of NK formalism is the choice of discrete rather than continuous strategy spaces. This choice was natural in the original applications of NK machinery to interactions among genes in biological systems (Kaufmann 1993). Its extension to all organizational choices may require more careful consideration however. There are organizational choices such as the selection of a market or the choice of a product architecture that fit the discrete conceptualization well. However, many other choices, from pricing to employee hiring, salary, training, allocation of organizational resources to various activities and investments in capabilities are better seen as continuous. In fact the cumulative nature of organizational resources and capabilities have long been seen as a source of complexity in building and sustaining them (Dierickx and Cool 1989). Interestingly, discrete vs. continuous organizational choices are loosely correlated with the distinction between external vs. internal fit (Siggelkow 2001). Specifically, the
external fit between an organization’s chosen position in the market and its internal strategy can be conceptualized as a discrete matching problem. Once that positioning question is settled, the internal fit among various organizational choices to serve the targeted market segment results to a large extent (but not exclusively) from continuous tradeoffs in allocation of attention and resources (Barney 1991, Burgelman 1991, Helfat and Peteraf 2003, Ocasio 1997). Strategy includes choices informing both internal and external fit, but separating the two is useful in thinking about implications for organizational heterogeneity. For example, the fact that a high-end restaurant has a different organizational structure from a McDonald’s branch is not surprising. It is heterogeneity among organizations targeting the same market that requires a theoretical explanation. That question better aligns with choices determining the internal fit, many of which are continuous. Therefore in the current paper we focus only on exploring performance landscapes in targeting the same market, leaving aside the performance tradeoffs related to market selection. There are also technical reasons to more seriously explore continuous strategy spaces. First, any discrete space is a subset of a continuous space, so what we learn on continuous spaces may be more broadly informative than insights from discrete spaces. Second, formulating continuous performance functions gives us access to tools from calculus available only on the continuous spaces. Therefore, it may be both conceptually worthwhile and analytically productive to explore interdependence on continuous strategy spaces and whether NK’s basic intuitions carry over in such an extension.

A second intuition that has accompanied, and may have been reinforced by, NK formalism is the association between complementarity among organizational choices and the ruggedness of performance landscapes. There is significant evidence for complementarity among various organizational choices (Ennen and Richter 2010, Milgrom and Roberts 1995). For example higher compensation, training, use of teams in organization of work, employee empowerment, and more customization of product offerings have been observed to coexist and reinforce each other in various manufacturing operations (Ichniowski et al. 1997, Macduffie 1995) and together offer a distinct alternative to the more traditional mass production (Womack et al. 1991). More broadly, empirical estimation of interactions among a variety of
choices have provided evidence for complementarities in diverse organization settings (Ennen and Richter 2010). In introducing the NK landscape to organizational audiences, Levinthal used the evidence for complementarities among choices to motivate the complex interactions embedded in the formalism (Levinthal 1997). Given that NK shows that interactions underpin ruggedness of landscapes, a significant subset (but not all; there are many notable exceptions see e.g., Rivkin (2000) and Porter and Siggelkow (2008)) of later writings on the topic have assumed or implied that complementarity leads to proliferation of local peaks (e.g., Dosi and Marengo 1999, Lee et al. 2010, Levinthal 2000, 2011, Matsuyama 1997, Narduzzo et al. 2000). This intuition, however, has not been tested rigorously, potentially because measuring complementarities in NK formalism is not straight forward. Given that the connection between complementarity and ruggedness is one of the few links between NK models and empirical regularities about organizational interactions, a formal test of this hypothesis is long overdue (in fact Porter and Siggelkow (2008) refer to unpublished results that do not support this intuition). Extending the NK landscape to continuous strategy spaces can facilitate such a test, because it is easy to measure and control complementarity (i.e. mixed partial derivatives of performance with respect to two choices) in continuous functions (Siggelkow 2002b).

Alternative formalizations of interdependencies among organizational choices not only informs the robustness and boundary conditions of the intuitions accompanying NK formalism, but also may help bridge between the economic and complexity views of organizations. Extending the NK formalization to continuous landscapes provides a path to realizing these opportunities. Specifically, I start by proposing a performance landscape applicable to continuous (as well as discrete) choices. This landscape is designed to include the NK formulation as a special case and be simple (in that it replicates the NK landscapes with the minimum number of free parameters). I then analyze this landscape to:  

A) study how the number of local peaks changes as a function of N and K and how that compares to NK results;  
B) analyze implications for search and size of basins of attraction;  
C) formally test the intuition that complementarity increases the number of local peaks; and  
D) explore how the landscape may evolve over time as a result of innovation, and how that evolution changes its topology over time. I conclude by discussing the
implications for our understanding of performance landscapes in systems of interdependent activities, role of management in understanding heterogeneity among firms, sources of complementarity among choices, and the bridge between this view and the more simple landscapes espoused by economic models.

MODEL

The idea that firms consist of interdependent activity systems (Fleming 2001, Porter 1996, Siggelkow 2011) implies that the contribution of each choice to overall organizational performance may depend on other choices. The NK model uses one operationalization of such interdependences: the contribution of each choice is unique and independent for any of the $2^K$ combinations of other K choices with which the focal choice interacts. While there is broad support for interdependencies among organizational choices (Ennen and Richter 2010, Milgrom and Roberts 1995, Siggelkow 2011), empirical assessment of this specific operationalization has been lagging (Bauman et al. 2019) and many alternatives could be conceived. A natural one is to formulate separate contributions to performance emanating from each interaction among choices informing organizational performance (Porter and Siggelkow 2008). The strength of each contribution would depend both on the value of interacting choices, and a coefficient regulating the sign and magnitude of the interaction effect. For example, the contribution of a 3-way interaction between choices 1, 3, and 7 to performance can be captured in a term $c_{s_1s_3s_7}$ where $c$ is a constant coefficient. Adding up these contributions over the relevant interactions generates the overall performance function. Given the use of interaction polynomials in building this performance landscape I call it PN and formally define it as:

$$\pi_{PN}(s) = c_0 + \sum_{i=1}^{2^{N-1}} c_i \prod_{s_j \in P_i} s_j$$

Here $P_i$ identifies the $2^N - 1$ non-empty subsets of the set $[s_1...s_N]$. Every potential interaction among choices is captured here: there are $\binom{N}{1}$ terms for the direct impact of each choice, $s_i$; $\binom{N}{2}$ terms for binary interactions $(s_is_j)$; and $\binom{N}{k}$ terms for k-fold interactions among elements of the organization’s strategy. The performance landscape is fully defined with the specification of coefficient vector $c$ (set of
$c_i$ coefficients $0 \leq i < 2^N$. $c_i$’s could follow a distribution (as the payoff contributions in NK do; here we draw $e$ from $\text{Uniform}(-c_{\text{max}}, c_{\text{max}})$) or be estimated to match specific characteristics of a problem. A PN landscape of interaction level $K$ can be defined as one with no contribution (zero $c_i$ values) for $P_i$ subsets of size larger than $K+1$. Equation 3 shows an example of PN performance function with $N=3$ and $K=2$:

$$\pi_{PN}(s) = c_0 + c_1s_1 + c_2s_2 + c_3s_3 + c_4s_1s_2 + c_5s_1s_3 + c_6s_2s_3 + c_7s_1s_2s_3$$  \hspace{1cm} (3)

While the binary interactions are commonly captured in empirical studies (Ennen and Richter 2010), higher order terms represent synergies or negative interactions among bundles of activities (Macduffie 1995). Finally, different ranges for organizational choices could be considered. For the analysis at hand we use $0 \leq s_i \leq 1$ to stay close to NK formulation, noting that most results remain unchanged as long as all choices are limited to some lower and an upper bounds.

**A few properties of PN landscapes**- The PN landscape has a few attractive features for the purpose at hand. First, it can easily apply to continuous choices as well as discrete ones: the $s_i$ values could be continuous or discrete, and the resulting performance can be consistently calculated on the subspaces of continuous $\mathbb{R}^N$. For example consider discrete computing platforms varying on three choices of $s_i$: Interface (Touch vs. Keyboard), $s_2$: Processor (Quantum vs. Conventional), and $s_3$: Algorithms (Qubit-based vs. Digital). With binary choices this example is well fitted to an NK framework, but can also be captured using the PN formalization (e.g. equation 3). In such a set-up, $c_2$, for example, would represent the relative contribution of Quantum (vs. Conventional) processors and $c_6$ will reflect the performance gain due to the match between processor and algorithms. Second, the performance function is continuous with well-defined derivatives with respect to various choices, making it easy to calculate complementarities among choices and to use continuous space analytical tools. Third, having explicit interactions (rather than differences in random draws informing interactions) improves the correspondence of the landscape to empirical estimations of interactions. As such, this formulation more closely tracks qualitative intuitions for substitution and complementarity effects.
Two other features of PN landscapes enhance their parallelism with NK formulation making PN a useful comparative device. First, the PN landscape can replicate any NK landscape while introducing the fewest number of free parameters. The basic intuition is that the vector $c$ has the exact number of parameters needed to match performance of any given NK landscape in corner points of strategy space. For example with $N=3$ the NK landscape is defined by 8 performance values for $2^3=8$ corners of unit cube. The PN landscape in eq. 3 has 8 parameters ($c_0$ to $c_7$) which can be tuned to generate the same payoffs as any randomly generated NK. Appendix A provides technical details. Note that while PN can replicate any NK landscape, there is no one-to-one correspondence between the two: NK formalism cannot replicate every PN landscape, and a randomly drawn PN landscape may be far from any NK one.

Second, similar to NK formulation, the local peaks in PN are all corner solutions, further facilitating comparisons between the two. In the NK landscape local peaks are points from which no change in a single choice improves performance. In a continuous strategy space interior local peaks can be distinguished from corner maxima. The former emerges where a point within the feasible region outperforms neighboring points in every direction (i.e. a point with zero gradients and negative-definite Hessian matrix on a smooth landscape), whereas corner maxima are only required to outperform feasible neighboring points. Appendix B details why PN peaks are all corner solutions, the basic intuition is that PN formulation, having no second/higher order terms for any choice, excludes tradeoffs within a given choice. Thus, in using PN landscapes to extend NK to continuous strategy spaces no new (interior) peaks are generated. Of course one may seek to introduce interior peaks for purposes other than comparisons with NK intuitions. Such formulations require additional terms (beyond the minimum required to replicate NK landscapes) that create tradeoffs within different levels of a given choice; I briefly return to this possibility in the robustness section.

RESULTS

Considering the key intuitions accompanying NK landscape in light of the PN generalization provides a few insights.
Growth in the number of peaks- Given that PN formulation can replicate NK landscapes, and that all the local peaks in both formulations are corner solutions, one may expect the number of local peaks in both to be in the same range and changing similarly with N and K. To test this hypothesis I measure the number of local peak under NK and PN formulations for different N and K values. For each N and K combination the analysis starts with generating 500 NK landscapes. I then solve for the corresponding PN landscape for each NK instance and find $c_{\text{max}}$, the largest (in absolute value) $c_i$ in that solution. Next, a comparison PN landscape is generated in parallel with each NK instance by drawing the $c_i$ parameters uniformly within the range $[-c_{\text{max}}, c_{\text{max}}]$. This procedure ensures that we simulate PN landscapes that are generated from the parameter ranges encompassing NK alternatives, but otherwise assumes little structure in their underlying coefficients, increasing the likelihood of seeing the range of outcomes feasible under PN formulation.

Figure 1 shows the results. The number of peaks grows with $N$ (values between 6 and 14 are simulated) and $K$ under NK (solid) and PN (dashed) landscapes. The graphs also include 90% confidence bands for each $N$ and $K$ combination. As already shown in the prior literature (Rivkin 2000, Rivkin and Siggelkow 2007), the number of local peaks in the NK landscape grows very rapidly with $N$ and $K$, reaching an average of 1093 with $N=14$ and $K=13$. Prior discussions of actual activity systems suggest that $N$ is large, e.g. Rivkin and Siggelkow (2007) identify over a dozen cases with $N$ varying between 13 and 111. Therefore local peaks would proliferate and the combinatorial complexity makes it extremely difficult to find the global peak, thus favoring the use of local adaptation as the only behaviorally realistic learning process. These results, however, are more subtle in the case of PN generalization: 1) For a given $N$, the number of peaks in the NK landscape is far larger than in the PN alternative and the gap between the two grows exponentially with $N$; for example average number of peaks in PN is only 27 with $N=14$ and $K=13$. 2) The growth in the number of peaks with $K$ is much slower in the PN formulation than in the NK and may become zero for higher $K$ values. This results is not necessarily surprising because the meaning of $K$ in PN is not identical to NK. Specifically, $K$ values in PN specify the depth of interactions (i.e. up to how many choices may interact with each other), whereas in the NK definition $K$ specifies the
number of other choices on which the contribution of each choice depends. In the PN formulation interactions’ impact on ruggedness saturates quickly and very deep interactions have little impact on the number of peaks. Therefore, even when all possible interactions are included, the gap between NK and PN formulations grows with \( N \). Curiously, for the \( N \) values we explored, the maximum number of peaks in the PN landscape can be accurately (\( R^2=0.999 \)) estimated by the random NK landscape with the same \( N \) and \( K=1.56\ln(N)-1.52 \) (i.e. \( K=1.3-2.6 \) in our simulations; peak numbers for non-integer \( K \) values are based on linear interpolations). In short, the ruggedness observed in NK for \( K>1.6\ln(N) \) is not likely to be seen in the PN landscapes.

![Figure 1](image_url)

Figure 1-Average and 90% outcome range for the number of peaks with different \( N \) and \( K \) values under NK landscape (Solid blue) and PN generalization (dashed black).

Previous results focused on the canonical NK formulation with random interactions, and the comparative results are qualitatively robust to considering other structured interaction patterns in the NK model. Table 1 compares the number of peaks in PN formulation against various patterns of interaction previously studied for NK formulations (replicated from Table 3 in (Rivkin and Siggelkow 2007); available for the case of \( N=12 \) and \( 1 \leq K \leq 6 \)). The highlighted columns report data from the current analysis. Replication of random NK closely matches prior results. While the gap between Dependent and Centralized interaction patterns with NK assumptions is large (others are in a closer range), the number of peaks in PN remains fewer than all others (except for low \( K \) values with Centralized NK), and grows...
much more slowly with $K$; in fact the gap would widen if larger $K$ and $N$ values were examined. In short, the impact of PN formulation on the number of local peaks exceeds the variance due to diverse interaction patterns within NK framework. Future analyses could consider different structures of interaction in PN formulation as well, though keeping it parallel to NK interaction networks may not be easy. For one thing conceptualization of interactions is not the same between PN and NK formulations; moreover, smooth and continuous functions exclude asymmetric interactions (because $\frac{\partial^2 \pi_{PN}}{\partial s_i \partial s_j} = \frac{\partial^2 \pi_{PN}}{\partial s_j \partial s_i}$).

Table 1- Comparison of number of peaks between PN and NK (results from (Rivkin and Siggelkow, 2007)) with different interdependency patterns ($N=12; 1 \leq K \leq 6$)

<table>
<thead>
<tr>
<th>K</th>
<th>NK</th>
<th>PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.7</td>
<td>7.6</td>
</tr>
<tr>
<td>2</td>
<td>133.3</td>
<td>21.6</td>
</tr>
<tr>
<td>3</td>
<td>177.1</td>
<td>43.6</td>
</tr>
<tr>
<td>4</td>
<td>209.5</td>
<td>74.2</td>
</tr>
<tr>
<td>5</td>
<td>238.2</td>
<td>114.4</td>
</tr>
<tr>
<td>6</td>
<td>248.7</td>
<td>100.3</td>
</tr>
</tbody>
</table>

What mechanism explains the significant gap in the number of peaks between the two formulations? The key factor is the difference in how interactions are conceptualized, rather than the patterns of interdependency among choices. Specifically, in the NK formulation a change in any interacting choice changes the contribution of the focal choice to a completely new, random, value. In contrast the payoff changes in PN are more gradual. Consider the example of discrete computing platforms varying in their Interface (Touch vs. Keyboard), Processor (Quantum vs. Conventional), and Algorithms (Qubit-based vs. Digital) with $N=3$ and $K=2$. In the NK formulation the observation that the Touch-Conventional-Digital platform has a large performance score says nothing about the expected performance from Keyboard-Conventional-Digital configuration. In fact even if we knew that the contribution of Conventional processor was very high in the first configuration (i.e. in combination with Touch interface and Digital algorithms), we could infer nothing about the contribution of conventional processor in combination with Keyboard and Digital algorithms). Essentially, in NK formalization
interactions inject heavy doses of randomness into the payoffs; at the limit, interactions are understood as chaos, creating completely independent random performance measures for every point on the landscape.

In contrast, interactions in the PN formulation preserve opportunities for inference of performance among related configurations. For example, the performance of Touch-Conventional-Digital is determined by fixed contributions of Processor, Algorithm, and Interface; interaction terms for Interface*Processor, Interface*Algorithms, and Processor*Algorithms; as well as their 3-way interaction. Therefore the observation that Touch-Conventional-Digital is high-performing would imply higher than average expected performance for Keyboard-Conventional-Digital, because the latter shares two fixed effects (for Conventional processor and Digital algorithms) as well as an interaction (for Processor*Algorithms) with the former. Thus, even with maximum interaction, neighboring configurations remain informative about each other, after all they share most choices and thus many interactions. Interdependence is not chaos, rather, multiple interactions that add up. Of course one can fine-tune the contributions of those interactions \(c_i's\) so that they exactly cancel each other out making neighboring configurations completely independent (that is why PN can replicate any NK outcome). However, such combinations of coefficients are unlikely to be observed by chance. In replicating the NK landscape using PN formulation we sample heavily from a tiny region of parameter space not otherwise visited in typical PN realizations.

The conceptualization of interaction embedded in PN formulation may be the more familiar for many organizational scholars. For example most empirical research on interactions among elements of firm strategy uses a variant of this conceptualization (Ennen and Richter 2010). This is not to suggest that the alternative embedded in the NK model is not applicable; rather, these conceptualizations may fit different empirical phenomena. The value of current analysis is in highlighting the difference between the two which sharpens the boundary condition for prior NK results. To the extent a result about adaptation,
local search, or competition using NK machinery depends on the large and exponentially growing number of local peaks, it may not fully generalize to alternative conceptualizations.

**Local search and basins of attraction in continuous strategy spaces** Expanding strategy spaces to include continuous choices has a few implications for search and its outcomes. These include implications for both the nature of search and the basins of attraction realized as a result of local adaptation. Whereas search on strategy spaces with binary choices is limited to random flipping of choices to find better alternatives, on a continuous strategy space the topology of local neighborhood, such as the gradient and curvature, can inform the directionality of search. In organizational settings both cognitive search (Gavetti and Porac 2018) and small daily variations experienced by diverse actors enable the formation of reasonable mental maps of local topology, even if actors do not have a global understanding of the landscape. That will allow them to focus on the more promising directions of local adaptation (though randomness and delays complicate this picture in practice (Levinthal and March 1993, March and Olsen 1975)), akin to gradient-descent optimization methods (Koziel and Yang 2011). On the other hand search on continuous spaces also requires the specification of step sizes, which is not trivial: small steps make search slow and inefficient, large steps can overshoot and be disruptive.

The introduction of PN landscape may also impact the size of basins of attractions, an important consideration in analyzing local adaptation. A landscape may have many local peaks but if one peak has a basin of attraction significantly larger than all other local peaks, then a large fraction of organizations are likely to start their exploration from within that peak’s basin of attraction. Therefore local search will lead most firms to a single peak and reduce the heterogeneity in configurations; outcomes that are qualitatively similar to those on a unimodal landscape. On the other hand when local peaks have basins of attraction of comparable sizes, the proliferation of peaks significantly increases heterogeneity. On the NK landscapes basins of attraction are generally of comparable sizes (Rivkin 2000). In other words, on NK landscapes not only many peaks exists, but most of them are relevant to adaptation to heterogeneous configurations. However, we do not know if the basins of attraction are more generally of comparable sizes, and thus if the number of local peaks provides a good window into heterogeneity outcomes. The PN formulation
provides an opportunity to test the sensitivity of this property to two mechanisms. First, given a discrete
strategy space with binary choices, the size of basins of attraction for peaks in the PN formulation may
follow a different distribution than NK. The lower the variance of that distribution, the higher is the
relevance of local peaks to heterogeneity. Second, PN allows us to consider continuous strategy spaces,
which may also change the size of basins of attraction. Specifically, prior discrete characterizations of
basins of attraction focused on the number of discrete configurations from which local search takes the
organization to a given peak. On PN’s continuous strategy space the basins of attraction are specified as
closed subsets of $\mathbb{R}^N$, and their volume represents their size. Given the same PN landscape (a standard one
or one replicating an NK landscape) it is not clear whether the discrete and continuous basins of attraction
would have similar size distributions. Therefore we don’t know whether considering continuous choices
in local adaptation increases, or decreases, the heterogeneity of local peaks discovered (compared to
discrete strategy spaces). These two mechanisms are explored next.

For this analysis I track the local adaptation outcomes of a population of firms under four
different experimental conditions for different values of $0 \leq K \leq 9$ when $N=10$. The experimental conditions
change the landscape definition (PN vs. NK) and continuity of choices (binary vs. continuous). For each
experimental condition, and $K$, the experiment is replicated for 500 different landscapes (and populations
of 500 firms each) to provide statistically reliable results. In each replication the 500 organizations are
placed randomly on the strategy space, search locally, and find various local peaks with ‘market shares’
($h_i$ for peak $i$) that in expectation are proportional to the size of basins of attraction for those peaks. The
concentration of $h_i$, formalized using Herfindahl index (Hirschman 1964), provides a simple measure to
summarize the results (Figure 2). Appendix D provides additional implementation details. For each
frequency, $F (>1)$, of discovered local peaks using local adaptation, we graph the Normalized Herfindahl
index: $H_N = \frac{F}{\sum_{i=1}^{F} h_i^2 - 1}$. This score varies between 0 (basins of equal size) and 1 (a single dominant
basin). Comparing these concentration graphs across the four conditions (Figure 2) reveals two features.
First, for a given $F$, peaks in the PN formulation are more likely to have unequal basins of attraction. That
is, for both discrete and continuous strategy spaces PN landscape (dotted line) has higher $H_N$ values than NK. The heterogeneity in size of basins of attraction in PN is mainly due to the higher correlation among neighboring points in this formulation, which boosts the size of larger peaks. The effect weakens as the number of peaks grows, and for the largest number of peaks observed in PN ($\sim F=10^{1.2}\approx16$) disappears.

Second, compared to discrete spaces, a continuous strategy space increases heterogeneity in the size of basins of attraction. This effect is robust to the choice of PN vs. NK formulation. To understand this effect, consider a two dimensional strategy space (unit square between 0 and 1) with 4 corners and two local peaks at two opposite corners. The discrete search will give each peak a basin with 50% share. In the continuous space basins are two non-overlapping areas that together cover the unit square. Except for singular cases, the size of these basins will be different, giving one a larger share. More generally, whereas local search is limited to changing one dimension at a time in discrete choices, which guarantees each peak a basin of at least N points, continuous space expands the larger basins of attraction by allocating a bigger fraction of intermediate space to the taller peaks and shrinks the size of smaller basins. This effect is pronounced for landscapes with modest ruggedness and attenuates with larger $F$ values. Overall, given a ruggedness level, these two effects may reduce heterogeneity emerging from local adaptation, especially in landscapes with a handful of local peaks. For example, consider landscapes with $F=4$ peaks. The discrete NK has a normalized Herfindahl index of 0.08 (e.g. an index resulting from market shares $h=[0.43, 0.28, 0.18, 0.11]$) whereas continuous PN has $H_N=0.47$ (e.g. resulting from $h=[0.75, 0.19, 0.05, 0.01]$); in the latter example the large majority of firms converge to a single peak.
(with its basin occupying 75% of strategy space) and very few (6%) land on the two smaller one. In contrast those peaks with smaller basins will have a notable 29% share in the discrete NK case.

Does complementarity lead to ruggedness? The empirical motivation for rugged performance landscapes has come from the observation that strategic choices are often complementary (Milgrom and Roberts 1995). As a result complementarity is seen by many as a source of ruggedness in performance landscapes (e.g., Dosi and Marengo 1999, Lee et al. 2010, Levinthal 2000, 2011, Matsuyama 1997, Narduzzo et al. 2000). This idea, however, has not been formally tested, as prior research has not measured levels of complementarity in NK or other formalizations of performance landscapes (See Porter and Siggelkow 2008 for an exception). I test this hypothesis in two steps, starting by an analysis of PN landscapes with the maximum levels of complementarity, i.e. supermodular ones. A supermodular landscape is one in which performance has a positive mixed partial derivative with respect to every two choice and across the strategy space, i.e. every two choice are always complements. While a rather restrictive assumption, supermodular systems have special properties that facilitates more clear insights (Milgrom and Roberts 1995, Topkis 1978). Moreover, as the case with highest levels of complementarity they offer an extreme condition for addressing the impact of complementarity on the number of local
peaks. In the second step I investigate the association between complementarity and number of local peaks in the more general cases of NK and PN landscapes.

Table 2 - Number of peaks in supermodular PN landscapes. For each $N$ equal number of landscapes with different values of $1 \leq K < N$ are simulated, but given limited impact of $K$ those results are combined for each $N$. Simulation cost grows exponentially with $N$, therefore fewer landscapes are simulated for $N > 8$. See appendix C for details on generation of supermodular PN landscapes.

<table>
<thead>
<tr>
<th># PEAKS</th>
<th>N</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>89.78%</td>
<td>97.90%</td>
<td>99.59%</td>
<td>99.92%</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>9.97%</td>
<td>2.07%</td>
<td>0.41%</td>
<td>0.08%</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.25%</td>
<td>0.03%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0</td>
<td>2e-3%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 Error! Reference source not found. reports the number of local peaks on simulated supermodular PN landscapes with different $N$ (and $K$; for each $N$ all $1 \leq K < N$ are included but variations in $K$ have little impact). A rather surprising result emerges in supermodular PN landscapes: regardless of $N$, $K$, or specifics of coefficients, such landscapes rarely have more than two local peaks. Moreover, even the bimodal landscapes become rare as $N$ grows. In regular PN landscapes the number of peaks does not grow as fast with $N$ as is the case with NK, nevertheless, dozens of local peaks are commonly seen for larger $N$ values. Why would the number of local peaks shrink so much when complementarities prevail? And why would the prevalence of unimodal landscapes increase with $N$? For an intuition, consider the derivatives of performance with respect to each choice. Loosely speaking, the change in the signs of these derivatives is required for changes in the basins of attraction and is therefore associated with the number of local peaks. These derivatives may be positive or negative at the origin, i.e. the point $s=0$. The second derivative for each choice $(\frac{\partial^2 \pi_{PN}}{\partial s_i^2})$ is zero (PN includes no second-order terms; see equation 2), therefore the first derivative only changes as a result of changes in other choices, i.e. the partial derivatives $(\frac{\partial^2 \pi_{PN}}{\partial s_i s_j})$.

Supermodularity dictates that those partial derivatives are non-negative throughout the landscape. Therefore the first derivatives can only increase as we increase different choices, with maximum derivatives found on the corner point with the largest values of choices (i.e. $s=1$). If the derivatives for some choices are positive at $s=0$, or negative at $s=1$, then those derivatives do not change sign throughout the landscape and the value of their corresponding choice in any local peak should be one, or zero,
respectively. Excluding those choices already reduces the dimensionality of choice-set that determines the ruggedness of landscape. The performance derivatives of remaining choices change sign (only) once in moving from \( s=0 \), to \( s=1 \), and depending on their interactions they usually create either two strictly ordered local peaks (a common property of supermodular systems (Milgrom and Roberts 1995)), or just one. Moreover, as \( N \) grows, the coexistence of two local peaks becomes increasingly unlikely as the peaks with the lower values for \( s \) vanish: with higher \( N \) there is likely a subset of choices with positive derivatives at \( s=0 \). These choices should be set to 1 in any local peak, moving potential local peaks to regions of strategy space with higher choice values. But setting them to one increases the derivative for other choices (due to complementarities), and potentially flips those other derivatives positive, which tends to push the landscape towards one with a single peak at \( s=1 \). In short, each new complementary choice strengthens the existing bundle of choices, increasing the advantage of the resulting peak and reducing the likelihood that any other peak remains on the landscape. In fact given that \( NK \) can be seen as a special case of PN landscape, in the unlikely case that by chance one generates a supermodular NK landscape, its number of local peaks, except in rare cases, will remain below three and will be declining with \( N \). Supermodularity is a strong assumption. Our second analysis provides more general evidence supporting the negative association between complementarity and number of local peaks. In the PN landscape the complementarity between every two choices can be measured for any point on the strategy space. Given the restrictive nature of supermodularity condition I use a continuous measure of complementarity, \( L \), defined as the average number of complements over 100 random points within the strategy space, divided by the maximum possible \((N^2-N)/2\). I assess, over 500 landscapes for each \( N \) and \( K \) combination (as reported in
Figure 1, there are 50 such combinations), whether this measure of complementarity predicts more peaks or not. Table 3 summarizes the results using regressions predicting number of local peaks with fixed effects for each $N$ and $K$ combination. Complementarity is an excellent predictor for having fewer peaks in the PN landscapes: a one-standard deviation increase in complementarity reduces the number of local peaks by 6.8. Interestingly the construction of NK landscape does not allow complementarity to vary by much. The $L$ measure ranges between 0 (no two choices are complements) and 1 (a supermodular landscape), with mean (standard deviation) of 0.45 (0.18) and 0.5 (0.03) for PN and NK respectively (measured across all $N$ and $K$ combinations). Moreover, the fixed effects for $N$ & $K$ combinations explain 99.8% of variation in the number of peaks (note the tight confidence band for NK results in
Figure 1 compared to PN). Therefore complementarity provides little additional predictive power in explaining the number of peaks in NK. Results are robust to other specifications including alternative measures of complementarity (continuous rather than binary counts of complements; not normalizing the number of complements to maximum possible; and inclusion of second order terms in regressions). Overall, evidence for complementarity does not correspond to ruggedness of performance landscapes; to the contrary, complementarity leads to smoother landscapes.

Table 3 - Impact of complementarity on ruggedness. Linear regressions. *** p<0.001, **p<0.01, *p<0.05.

<table>
<thead>
<tr>
<th></th>
<th>DV: # of Peaks in PN</th>
<th># of Peaks in NK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1 (0.27) ***</td>
<td>1.2 (0.95)</td>
</tr>
<tr>
<td>Complementarity (L)</td>
<td>-37.60 (0.40) ***</td>
<td>-0.52 (1.72)</td>
</tr>
<tr>
<td>Dummies for N &amp; K Combinations</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Observations</td>
<td>25000</td>
<td>25000</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.733</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Innovation and Evolving Landscapes

There is a large gap between the economic views on the topology of production functions and the alternatives, such as NK and PN, motivated by the interdependences among organizational choices. The gap is rooted in the different functional forms the two perspectives adopt in modeling performance landscapes: unimodal and concave vs. functions with many local peaks. To narrow this gap empirical studies can build the functional form of performance landscapes from more micro observations of
interactions in activity systems (Siggelkow 2001, 2002a). A theoretical approach can start with plausible assumptions on how landscapes evolve over time and examine the resulting topology. Here I leverage the PN formulation to facilitate an attempt towards the latter, theoretical, approach.

A starting point is the recognition that performance landscapes continuously change as a result of technological and organizational innovations (Henderson and Clark 1990) as well as shifts in the environment (Milliken 1987). Innovations may increase the efficiency of some activities, smooth out the interface among multiple activities, or introduce new activities all together, among others. Environmental shifts, from new regulations to changes in the underlying customer taste, and emergence of substitutes or complements also change the costs and benefits associated with various choices and their interactions. In effect, both environmental change and innovation can alter the landscape by changing the contribution of different choices, or their interactions, to overall performance. Within the PN formulation such changes can be formalized as mutations in the coefficient vector $c$.

There is, however, a key difference between landscape evolution induced by environmental shifts and change due to innovation. Environmental shifts are usually involuntary: organizations recognize those shifts and adapt to the new performance landscape. In contrast the adoption of emerging innovations is a choice made by organizations once an innovation is recognized. This choice usually depends on whether the innovation enhances organizational performance. The process of recognition and adoption of innovations takes time and could be complex. However, for our purposes one can abstract away from those complexities and distinguish between mutations that represent environmental shifts vs. those capturing innovations: the latter only change the landscapes if they are performance enhancing.

With these preliminaries in place, we model the evolution of PN performance landscapes in response to environmental shifts and innovation. Starting from an initial landscape, every period the type of change is first determined (an innovation with probability $p$ or an environmental shock with probability $1-p$). The landscape is then exposed to a mutation (a draw from $Uniform(-c_{max}, c_{max}); c_{max}=20$ allows for replicating NK formulations with $N=10$ which we simulate here) in one of the elements of the coefficient vector $c$. This mutation is always incorporated in the landscape when it reflects an environmental shift.
An innovative mutation is assessed for performance enhancement by the population of firms, and only then is incorporated in the landscape (i.e. the underlying innovation is adopted by various organizations changing the landscape they face). The innovation assessment and diffusion is a complex social process, so for the purpose here it is more productive to abstract away from those details and consider simple measures that inform which innovations are considered performance-enhancing by the population of organizations and thus incorporated in the landscape. One natural possibility considers a mutation performance-enhancing if it increases the average payoff across all local peaks on the landscape (See robustness section for a discussion of other alternatives). Figure 3Error! Reference source not found. summarizes the results for PN landscapes with $N=10$ and $K=9$, simulated over 3000 periods, and six values (between 0 and 1) for the relative frequency of innovative shocks ($p$). Panel A reports on the number of peaks over time, averaged over 500 simulations for each level of $p$. Panel B reports the changes over time in $L$, the average fraction of choices that are complements, a simple measures of complementarity (also used in regressions above).

The number of local peaks starts from around eight and goes down as the landscapes evolve in response to various shocks. This decline is a function of the parameter $p$, the propensity of innovative shocks. Not surprisingly, with $p=0$ we see no change in the number of peaks: mutations resulting from environmental shocks only replace a random coefficient with another coming from the same distribution, and in expectation do not change the topology of the landscape. On the other extreme, with $p=1$, all mutations are performance-enhancing innovations. Curiously, these mutations sharply reduce the number of local peaks to a single one. Innovations enhance performance by increasing the contribution of a single choice or positive interactions among multiple choices. In PN landscape such enhancements are typically realized when coefficients in $\epsilon$ are increased. The increase in interaction coefficients increases complementarity among choices (see panel B), which as we saw in the previous section, reduces the number of peaks significantly. Over time innovations wipe out intricate interactions that created local peaks, until a single global peak remains.
Naturally, the number of local peaks declines faster when innovative shocks are more common (high $p$ values). The steady state number of peaks is however determined in a race between the two types of mutations. Above a threshold in $p$ the innovative mutations ultimately take the landscape to a single-peaked topology. Below that threshold other equilibria are possible, ranging between one and the number observed for a random landscape. The interaction between the two types of shocks explains these results: innovative mutations are continuously adding complementarity, increasing the performance benefits of mutated interactions, and pushing the landscape towards a unimodal one. Environmental shocks, by injecting randomness, push the system back towards its original, more rugged, configuration. However, innovative mutations are more impactful: a single innovation can boost the value of one choice significantly and thus wipe out multiple local peaks. In contrast, injecting new peaks through random mutations is slow, requiring serendipitous changes in multiple interactions. The overall dynamic results from the balance between the two forces: as long as innovative mutations are stronger overall (given frequency and per-innovation impact), the system converges to a single global peak ($p$ values above 0.2 in our experiment). When environmental shocks are stronger, a balance is struck between the two forces leading to ruggedness levels spanning the two extremes ($p$ values between 0 and 0.2). Note that while emergence of complementarity is a major contributor to the generation of unimodal landscapes, it is not a
necessary condition; in earlier periods as well as steady state for $p=0.4$ complementarity fractions below one accompany unimodal landscapes.

Overall, evolution of performance landscapes as a result of innovation has a profound effect on the ruggedness of landscapes. By introducing complementarities and novel, more effective, choices/interactions, such innovations reduce the number of local peaks and over long time horizons may even lead to unimodal landscapes. In presence of innovative shocks a handful of local peaks may coexist, yet this number is likely smaller than what a random PN landscape indicates, which in turn is far below the NK counterpart. The broader relevance of these results partly depends on whether innovations increase complementarities more generally. That link is clear here: an innovation increases $c$ elements, i.e. finds a way to combine two or more choices more efficiently; assessing whether actual innovations have such complementarity-inducing property is an interesting area for further research.

Robustness and boundary conditions

The goal of this paper is to offer one data point on implications of interdependency among organizational choices for the characteristics of resulting performance landscapes and adaptation on those landscapes. A single paper cannot fully flesh out all those implications for a new formulation; even in the case of well-studied NK formulation over 70 papers have done so and more remains to be done. Therefore I focus the discussion of boundary conditions to factors relevant to preceding analysis.

When do interior peaks emerge? Earlier we saw that PN landscape includes no interior peaks. In practice interior peaks exist and reflect continuous tradeoffs managers regularly deal with. Including those interior peaks would require additional terms in the performance function to capture the relevant tradeoffs. A countless number of alternatives could be conceived, all requiring more parameters than the minimum required for keeping the parallelism between PN and NK formulations. In their simpler forms one could add the impact of second order terms (e.g. $\sum_{i=1}^{N} d_i s_i^2$ or $\sum_{i=1}^{N} d_i s_i (1 - s_i)$) with $N$ additional parameters ($d$). Such a term would potentially introduce new interior peaks, though in practice many of those peaks would not fall in the feasible unit hypercube, and thus may only reinforce existing corner peaks. Detailed analysis, reported in appendix E, shows that this extension will only increase the total
number of peaks or introduce a significant number of interior peaks when parameters \(d\) has a wide range (overwhelming the interactions due to original PN formulation), is designed to keep interior peaks feasible, and includes both positive and negative values (to induce both peaks and valleys). Such augmented versions of PN formulation enable considering interactions between interior and corner peaks though their detailed analysis is beyond the scope of current paper.

**Do innovative shocks always smooth the landscape?** The results on evolving landscapes show that innovative shocks smooth performance landscapes under a specific formulation. A few extensions assess the robustness of those results. First, major innovations may introduce completely new choices, rather than impacting a single interaction. To keep \(N\) constant, such change can be captured by redrawing all the interaction parameters of one of the choices; simulating such innovations finds no qualitative change in the basic result: innovative shocks will smooth the landscape, ultimately resulting in a single peak. Second, the previous analysis used the impact on the average performance of all local peaks to identify performance-enhancing innovations. Other measures could inform whether an innovation is performance enhancing. For example innovations that increase the performance of best strategy (global peak) may be particularly attractive among a population of firms prone to imitating the best-performing competitor. In a more egalitarian population, innovations may be adopted if they are seen as beneficial by the majority of firms given their current position, not only those who have landed on a local peak. These two ideas can inform two alternative measures of performance-enhancement: 1) A mutation is performance enhancing if it increases the performance of the global peak on the landscapes. 2) A mutation is performance enhancing if it increase the average payoff calculated over all (corner) points of the payoff landscape. The qualitative results remain unchanged to using both these alternatives.

Third, innovations considered earlier were rather incremental, in that they changed one interaction or one choice. Changing the size of innovative shocks, so that it ranges between single mutations to complete disruption of every interaction, shows that results extend to more radical innovations. The speed of convergence to a single peak varies, with innovative shocks of intermediate size offering the fastest convergence: single mutations have a good chance of adoption, but are slow in
accumulating to form a dominant peak; radical innovations changing all interactions are not very likely to be performance-enhancing, but those that are quickly reduce the number of local peaks (See Appendix F for details). Finally, one may be interested in whether these results extend to NK landscapes, or are limited to PN formulation. Intuitively, if an NK landscape is replicated using a PN formulation, and then exposed to innovations in the estimated $e$ vector, the results would be similar to regular PN landscape, only more dramatic. The initial ruggedness levels are much higher for NK landscapes and thus the drop to a single peak more pronounced. A more nuanced picture emerges when innovations are defined within the traditional NK formalism, as re-drawing the random contribution factors used for the construction of the NK landscape (See Appendix F). In that case innovations still reduce ruggedness, but not as fast as in the PN formulation, and are unlikely to lead to a single peak. Similar to the previous robustness test, the size of innovations also has a nonlinear effect on the NK results, with intermediate shocks having the largest smoothing effects.

**Do complementarities increase the cost of misperceptions?** In a 2-dimensional continuous and unbounded strategy space misperceiving complementary interactions is costly (Siggelkow 2002b). The location of (interior) peaks is sensitive to strength of complementarities, and thus misperception of those strengths will lead actors astray. Given the emergence of complementarity in evolving PN landscapes, and the impact of complementarity on reducing the number of local peaks, that mechanism may be especially relevant to PN landscapes. To analyze this possibility I assess cases where perceived interaction pattern is different from true interactions, $e$, and organizations seek the global peak based on their perception. I then measure how the performance and location of global peak is impacted by complementarity and its interaction with misperception (details reported in Appendix G). This analysis does not find the significant effect of misperceived complementarities previously reported, in fact, complementarities have a beneficial interaction with misperceptions. This outcome is best understood in light of the bounded strategy space in the PN landscape, compared to unbounded strategy spaces leading to interior peaks (Siggelkow 2002b). In an unbounded strategy space with interior peaks, even the smallest misperceptions of complementarity changes the location of interior peak. When peaks are corner solutions, a change in
the (perceived) global peak requires notable misperceptions along multiple interactions, otherwise the perceived and actual global peaks may remain the same. On the other hand, prior results focused on the case with a single peak, thus not accounting for the impact of complementarity on reducing ruggedness. On the PN landscape increased complementarity reduces the number of local peaks and thus increases the robustness of remaining peaks to misperception: with fewer competing alternatives the chances are that the global peak will not change as a result of minor misperceptions.

**DISCUSSION**

Performance landscapes -- the mappings from managerial choices to organizational performance -- are at the heart of understanding strategy and heterogeneity in organizational forms and outcomes. Nevertheless, the two competing schools in formalizing performance landscapes have diverged. While the one rooted in economic theorizing starts with unimodal, concave production functions, the NK alternative promotes landscapes with a multitude of local peaks. In parallel to NK theorizing I introduce the PN landscapes that views firms as systems of interdependent activities (Porter 1996, Siggelkow 2011). This alternative explicitly models every possible interaction among choices, applies to both continuous and discrete choices, includes NK as a special case, and is used as a vehicle to assess the robustness and boundary conditions of broad intuitions accompanying prior NK literature. While PN stays close to NK in only including corner peaks, simulation results reveal significant differences between the two. The growth in the number of local peaks as a function of $N$ and $K$ is much slower in PN compared to NK. Fixing the number of peaks, basins of attraction are more unequal in continuous PN landscapes. Moreover, complementarity, which some prior work has postulated as a driver of ruggedness, turns out to be a major mechanism for smoothing out the landscapes. In fact, supermodular PN landscapes rarely have more than two local peaks and that number declines with $N$. Furthermore, performance enhancing (innovative) mutations tend to increase complementarity and reduce the number of peaks, often to a single one. These findings have important implications for our understanding of performance landscapes, heterogeneity in organizational forms and outcomes, role of management, and complementarity.

**Bridging different understandings of performance landscapes**
A large gap has persisted between economic conceptualizations of performance landscapes and those rooted in interactions among elements of activity systems. This gap is most clearly manifested in the different number of local peaks in typical performance landscapes conceptualized by each community: whereas the economic production functions often assume unimodal landscapes, the NK formalization may result in a plethora of local peaks. The wide chasm between these views has limited the dialogue among the relevant research communities. The gap also forces researchers to choose between incommensurate alternatives without clear guidelines informing the appropriate option for a given phenomenon. Current study may reduce this gap in two ways. First, capturing interdependencies among elements of activity systems does not necessarily lead to as many local peaks as previously conceived. The exponential growth in the number of peaks with \( N \) and \( K \) observed in the NK formulation is rooted in viewing interdependence as a harbinger of randomness. More continuous conceptualizations of interdependence create far fewer local peaks and are more consistent with the formulations used in empirical assessments of interactions among organizational choices. The gap between NK and PN landscapes grows with the dimensionality of strategy space (\( N \)), suggesting that taking interactions seriously is not as wildly inconsistent with the smoother landscapes economists typically invoke. Nevertheless, the PN landscape is still a rugged one with increasing number of peaks as \( N \) grows.

A second mechanism, evolution of landscapes with innovation, may bring the two perspectives even closer. Innovative shocks, by introducing more productive choices and interactions, create more distinct peaks with expanded basins of attraction that wipe out multiple other local peaks. Over time the landscape may evolve towards one with a single peak. In practice the emergence, recognition, and diffusion of innovations takes time, and thus actual firms are more likely competing on a transient landscape and not the one indicated by the steady-state results. We therefore may expect landscapes with a handful of local peaks to be a common occurrence, but one not emphasized by either the NK literature or traditional production functions. Prior theoretical results that rely only on the existence of multiple local peaks, but not on their large number, are robust but those depending on very large number of peaks may require careful examination. As a first order approximation, prior NK results found for \( K<1.6\ln(N) \)
are also consistent with the PN formalization. A more rigorous analysis could replicate prior work with the PN, capture landscape evolution, and identify which findings may be sensitive to these mechanisms.

Landscape evolution also provides a conceptual path to bridging between the two views of performance landscapes. Both the economic production functions and complex landscapes start with theoretical functional forms (e.g. Cobb-Douglas or NK) which *predetermine* the landscape topology and implicitly exclude intermediate alternatives. Explicitly modeling the evolution of landscapes in response to innovation allows us to start with the fundamental mechanisms of landscape change and let the topology emerge as a result. In this perspective the resulting landscape is less determined by our initial functional assumptions and more by the assumptions about landscape evolution. By focusing on the evolutionary mechanisms, for which we may have better process data and intuitions, we do not need to start with strong assumptions about final landscapes. In fact the evolving landscapes may prove relatively insensitive to initial topology. For example the evolutionary process we modeled would quickly lead to similar final configurations regardless of whether PN or NK was the starting point, even though the initial number of peaks are very different across the two.

**Heterogeneity among organizations and role of management**

Understanding the sources of heterogeneity among organizations has motivated scholars across multiple communities (Barney 1991, Levinthal 1997, Syverson 2011, Teece et al. 1997). The perspective researchers adopt on the topology of performance landscape, while sometimes implicit, significantly impacts the types of explanations they espouse for heterogeneity. Unimodal views of performance landscapes are common not only in economic production functions, but also in the popular management books and consulting and inform a large industry organized around disseminating best practices. On a unimodal landscape not only foresighted optimization but also naive learning can reliably locate the best strategy (Levinthal 2011); firms may adopt best practices with different speeds, but sustained heterogeneity among those targeting similar markets is a puzzle (Syverson 2011). The introduction of rugged landscapes (Levinthal 1997) has offered a potential explanation for this puzzle: facing many local peaks firms engage in local adaptation and not global optimization, and commonly converge to local
peaks rather than a single optimum strategy. While this explanation builds on the existence of multiple local peaks, it does not require the number of local peaks to be particularly large, as long as local adaptation is a plausible assumption. The number of local peaks, however, has two more subtle implications relevant for understanding heterogeneity. First, the dominance of local adaptation is partially motivated by the combinatorial complexity of finding the best strategy (Rivkin 2000). The more rugged the landscape, the harder it is to imitate successful examples (Rivkin 2000, 2001). It would also be harder to use cognitive maps to extrapolate from past experience and come up with new and promising strategies (Gavetti and Levinthal 2000). These complexities, in turn, strengthen the argument for local adaptation, which is the other necessary component for explaining heterogeneity based ruggedness of landscapes. If, as indicated by our results, interactions among components of strategy do not necessarily lead to an explosion in the number of local peaks, then imitation, replication, and cognitive search may offer more fruitful pathways to organizational search and pathways to convergence among competing organizations.

The second implication relates to understanding the role of management in strategy. Unimodal and very rugged views of landscapes disagree on many things, but they both see a limited role for managerial skills and capabilities. On the one hand, on a concave landscape both naïve search heuristics and rational managers are likely to find the best strategy, so the differential value of management quality is limited. On the other hand, if managers are seeking better strategies in presence of thousands of local peaks they are unlikely to develop from one set of experiences insights relevant for another scenario (Zollo 2009). Thus organizations will likely end up with very different strategies and outcomes, more as a result of luck, initial conditions, or superstitious learning than managerial capabilities. It is only in-between these two extremes -- when management is concerned with discovering the best among a handful of internally consistent viable bundles-- that managers’ role can be really important. In this middle-ground the underlying search problem would be neither too easy nor NP-hard. Management can then accumulate skills from prior experience, or observing others, which would be relevant on a new landscape. Moreover on less complex landscapes challenges to learning may fall into predictable patterns, such as capability traps (Repenning and Sterman 2002) and sunk cost fallacies (Arkes and Blumer 1985),
which can distinguish between quality managers and less effective ones. Those skills, or systematic biases, can then create a management-effect in firm strategy and performance (Bloom and Van Reenen 2007, Quigley and Hambrick 2015). Individual learning across a few stereotypical contexts may create systematic differences among individuals in the quality of their cognitive maps for a new problem and their facility in leading transitions from current organizational configurations to the desired ones. A limited number of local peaks would also be promising news for codifying and disseminating managerial knowledge. For example some typologies in popular management writing, such as high and low roads (McGregor 1960) and differentiation and cost leadership strategies (Porter 1998), have identified dichotomies that promise insights in finding successful strategies. While the usefulness of any such typology should be carefully assessed and debated, the idea of typologies becomes promising if one can delineate and quantify a handful of local peaks in any given application domain.

Results may also have implications for heterogeneity in organizational forms across industry life cycles. Specifically, landscape topology may endogenously change over the life of an industry and thus contribute to technology life-cycle patterns. Early in an industry not only the unpruned baseline interactions lead to a more rugged landscape, but also environmental shocks are more likely (e.g. new regulations (Milliken 1987), emergence of alternative categories (Durand and Khaire 2017), and institutions (DiMaggio and Powell 1991)). Firms competing on this initially rugged and turbulent landscape are likely to diverge in their strategies, consistent with the evidence for proliferation of competing organizational forms early in the industry life cycle (Abernathy and Utterback 1984). As the industry matures, product and process innovations outpace environmental shocks and create more smooth landscapes, potentially with a single peak manifest as a dominant design (Suárez and Utterback 1995). These regularities in industry life cycle have long been known (for a review see (Peltoniemi 2011)); the analysis here may offer a more fine-grained mechanism for the emergence of those regularities rooted in the evolution of performance landscapes in response to innovation.

**Performance landscapes and complementarities**
Some of the prior theorizing has suggested that complementarity among organizational choices leads to ruggedness of performance landscapes (e.g., Dosi and Marengo 1999, Lee et al. 2010, Levinthal 2000, 2011, Matsuyama 1997, Narduzzo et al. 2000). This view, however, has not been tested before and in fact some have contested the implied association (Porter and Siggelkow 2008). The analysis reported here shows that the association is in the opposite direction: complementarity significantly reduces the number of local peaks. The results are even starker for supermodular PN landscapes: when every two choice are complements, rarely more than two local peaks emerge. The two potential peaks in a supermodular landscape are typically very distinct, in that they fall on the two opposite extremes across multiple choices. This distinctiveness, which may for example be observed in differences between high road and low road strategies (Macduffie 1995, McGregor 1960), could predispose observers to extrapolate and expect more intricate complementarities to generate many distinct local peaks. However, the mathematics of complementarity do not support this intuition. In contrast, complementarity among two choices slants the landscape in favor of strategies that combine high values of both, and thus can wipe out local peaks at lower levels of those choices. This structure then significantly reduces the possibility that new optima are found by increasing one choice and reducing another (Milgrom and Roberts 1995), thus reducing the overall likelihood for emergence of local optima with complementarity.

On the other hand evolution of performance landscapes may inform the emergence of complementarities. Scholars have for long identified complementarities among organizational choices as both common and consequential in thinking about firm structure and strategy (Ennen and Richter 2010, Milgrom and Roberts 1995). The theoretical work on supermodularity (Milgrom and Roberts 1995, Topkis 1978) has been augmented by a large empirical literature documenting abundance of complementarities in measuring interactions among organizational choices. For example, a review of this literature found far more positive interactions (complementarities) than negative ones (substitutions) (Ennen and Richter 2010). Prior research, however, has not provided a more fundamental explanation for the emergence of such asymmetry: why should there be more positive interactions than negative ones? The analysis in this paper provides one possible explanation. Specifically, innovations that enhance
performance are more likely to also increase complementarity. Therefore, even if we start from random interactions (half complements and half substitutes), adopted innovations gradually turn substitutes to complements. These results provide a novel mechanism for explaining the existing empirical asymmetry between complementarity and substitution effects.

**Limitations and opportunities for future research**

This paper explored a few generic properties of performance landscapes but much remains to be done. The PN formulation as well as the framework for modeling the evolution of landscapes can be applied to a variety of prior problems explored using NK and related alternatives. However, PN landscape is not designed as a replacement to the NK platform, in fact, the reach and relevance of the results are limited by a few conceptual and empirical considerations. First, the PN landscape is one of many possible ways to conceptualize interactions that shape performance landscapes. It is designed to closely relate to the well-established NK architecture, and yet it generates results that depart rather significantly from some of the basic intuitions associated with NK models. This observation should give us pause in generalizing too widely based on PN results as well, after all one may find other plausible formulations of interactions that offer results different from both PN and NK. Second, a characteristic of both landscapes is the lack of interior peaks, more an artifact of mathematical functions used in generating each than the organizational phenomena these formulations are intended to capture. Interior peaks are often generated as a result of real world tradeoffs and nonlinear constraints. As discussed in the robustness section, those tradeoffs can be captured in formulations that include non-zero second derivatives with respect to some choices. However, generically adopting such formulations introduces many additional degrees of freedom in high-dimensional performance landscapes that complicate analysis and may blur theoretical insights. Ultimately, only so much can be learned from purely theoretical landscapes and it is not clear how much additional theoretical mileage can be expected from generic landscapes beyond what is already explored through NK formalism. Current literature can benefit from better connections between formal landscapes and actual interactions within organizational activity systems to guide our choices on the level of complementarity, existence of interior vs. corner peaks, the nature of interactions (e.g. random outcomes...
vs. continuous contributions), and dynamics of moving on performance landscapes. Progress on that agenda has been slow but can be informed by in depth case studies that map out relevant interactions in organizational activity systems (e.g., Cattani et al. 2017, Siggelkow 2001, 2002a) and then formalize those using plausible and robust functional forms. Empirical estimation can then be pursued with the aim of constraining the parameters of such models. Results not only would inform the shape of landscapes, but also can be used to analyze how the topology and dynamics of moving on performance landscapes conditions managerial learning, organizational adaptation and heterogeneity in diverse empirical settings.

REFERENCES


Kaufmann S (1993) *The origins of order*

Koziol S, Yang X (2011) *Computational optimization, methods and algorithms*


