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## **Problem-solving and Generality as Sources of Growth and Heterogeneity**

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### **Abstract**

The basic idea of this paper is that heterogeneity arises in a market because firms, meant as problem-solving entities, develop specific knowledge to rule out the bottlenecks and the technical challenges they have to face. The newly created knowledge is embodied in an innovation, and therefore an alternative explanation of the cause of innovative activity at the firm level can be found in the necessity to overcome problems, more than in the reaction to opportunities and incentives. Moreover, problem-solving activities can end up in different types of innovations, characterized by different "degrees of generality", which in turn affect firms' productivity. Fluctuations in productivity can therefore be the emergent result of many co-existing technological (capital vintages) dynamics. The setting of a simple model is finally proposed in order to summarize the theoretical building blocks introduced in the paper.

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## 1 Introduction

The empirical research in the field of industrial dynamics has highlighted a rich statistical structure holding along the evolution of industries, nicely fitting distributions of firm’s size, profitability, and productivity (Dosi, 2007). The persistency feature of the regularities seems to rely on complex-phenomena mechanisms (Kirman, 2010), where the statistical patterns are emergent properties, broken symmetries unfolding from the aggregation of micro behaviors. Stable structures at the industrial and macro level develop from a deep heterogeneous micro scenario, where the variability and the dispersion of individual characteristics doesn’t decline the more disaggregate the unit of analysis are. In addition to the heterogeneous distribution of characteristics and competencies among economic agents, the “shape”, the pace and the rate of the dynamics in an industry strongly depend on its degree of turbulence (Cantner, 2011), that is the regime of entry, exit and selection prevailing in a given period.

Market turbulence and individual heterogeneity affect each other, turbulence increasing the incentives and the constraints for firms to adopt specific behaviors and techniques, heterogeneity influencing the degree of fitness and success in interaction, meant both as economic competition and cooperation. The process outlined is evolutionary in nature<sup>1</sup>, thus suggesting that an explanation to relevant research puzzles and paradoxes such as “why firms differ” (Nelson, 1991) or “why growth rates differ” (Fagerberg, 1988)

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<sup>1</sup>This claim does not necessarily imply the generalization of Darwinian categories to economic phenomena; for the theoretical debate about the meaning of evolution and Darwinism in economics see, for example, Witt and Cordes (2007), Vromen (2010), Buenstorf (2006).

at the macro level has to be found in the sphere of economic dynamics. Turbulence, that is a form of restless change (Metcalfe et al., 2006), together with the emerging heterogeneity of economic actors, are the results of a process of differential growth, which in turn is strongly depending on the outcomes of the innovative activities undertaken by firms (Cohen and Levin, 1989).

The centrality of innovative activity as a determinant of differential growth is at the moment a consensus amongst appreciative theorists, but the forces acting behind the decisions to engage in innovative activities are still a matter of debate. As a result of the inconclusive inquire into the role of proximate causes such as firms' size and market structure (Cohen, 2010), the neo-Schumpeterian literature stresses two main reasons for innovative activity to occur, namely the exploitation of technological opportunities and the potential returns coming from economic chances (with particular attention to demand conditions), to which it should be added the inducement argument. In the latter case, innovative behavior follows the changes in factor prices or — in a more systematic approach (Antonelli, 2007) — is fostered by the mismatch between firm plans and actual conditions. The exploitation of technological opportunities relies both on economic and technical arguments to explain innovative behaviors, and captures nicely the stylized fact of a rich technological variety and dynamism of industries; nonetheless, specification and measurement difficulties on the empirical side (Levin and Reiss, 1984) and modeling complexities (Cantner and Pyka, 1998) have constrained its adoption outside the field of innovation studies.

The pure economic explanation of innovation incentives, that stemming from profit-maximizing behaviors, both in industrial organization non-tournament models (Dasgupta and Stiglitz, 1980) and in incumbent-entrants races (Reinganum, 1985), is by far the most relevant in the literature. The same holds for the modeling of technological change in macroeconomic (endogenous) growth models approaching steady-state equilibria. Two assumptions characterize these modeling strategies. The first concerns the economic incentive to reallocate resources to innovative activity (or, more in general, to Research and Development), which derives from the inter-temporal optimization of marginal returns in different activities or from the decision to participate in a race where the benefits of the monopoly right after an innovation are challenged by the threat of potential entrants in the market. The second assumption has to do with knowledge, which is usually considered to be additive and homogeneous. Knowledge generation and accumulation, in particular for what concerns growth models, are seen as the main forces at work behind innovation and growth in productivity, variety and quality; unfortunately, its characterization is far from being satisfying.

In this paper, I attempt to tackle several of the issues mentioned above. First, concerning the mechanisms leading heterogeneity to arise among economic actors, I suggest that an alternative explanation for innovative activity can be found looking at firms as problem-solving entities. From this perspective, innovations are meant as the outcome of a process of solutions search in order to solve technical and production problems. Innovations cannot be just an outcome of the optimal allocation of resources between alternative ends, if firms need them in order to survive in the market. Connected to this point is the dynamic of knowledges production. Conversely to the representation of knowledge in growth models, that of a generic stock uniformly affecting the economy, knowledge is inherently "directed", specific to firms and applications (Pavitt, 1984) and localized to deal with problems and with the search for solutions. The iterated process of "taylor-made" (Aghion and Howitt, 1998) knowledge production aimed to solve firm-specific problems is the driver of heterogeneity in this paper.

The second contribution of the paper comes as a corollary of the first point. The outcomes of the problem-solving activity in terms of knowledge production and technological innovation will affect firm's performances, and more precisely productivity and profits. In particular, a micro-level explanation of productivity's dynamics will be suggested based on a peculiar characterization of the process of technological change. In short, what is introduced is a differential impact of each innovation, in turn determined by the innovation's breadth of applicability, interpreted as the capability to solve larger or smaller sets of problems. This is realized via the introduction of an innovation-specific degree of generality, in turn inspired by the literature on General Purpose Technologies (GPTs, from now on).

In order to combine the building blocks mentioned in the paper, I will introduce the setting of a simple model, non-radical in its structure, but novel in its premises and aims. A final remark concerns the scale of analysis; the paper talks about firm's behavior, but is meant to represent micro-level relations capable to produce aggregate results. Therefore, I will refer both to growth and industrial dynamics literature, aiming at bridging those two research traditions.

The paper proceeds as follows. Section 2 deepens the discussion about the nature of knowledge, its representation and its role in directing the behavior of firms. Section 3 introduces the other building block of the analysis, namely the concepts of problem-solving and generality. Section 4 outlines the structure of the model to summarize the central tenets of the paper and to show their interaction. Final section concludes.

## 2 On the Directedness of Knowledge

The basic idea of this paper is that heterogeneity arises in a market because firms, meant as problem-solving entities, develop specific knowledge to rule out the bottlenecks and the technical challenges they have to face. Usually, the newly created knowledge is embodied in an innovation, and therefore an alternative explanation of the cause of innovative activity at the firm level can be found in the necessity to overcome problems, more than in the reaction to opportunities and incentives. In order to develop this argument, the claim that knowledge is largely firm specific has to be — at least briefly — defended.

Most of the literature accepts Arrow's (1962a) interpretation of knowledge (or, in Arrow's later terms, technical information (Arrow, 1996)) as a peculiar commodity, whose public good nature affects welfare and social optima. Conversely, the very nature of knowledge is much more complex and multifaceted<sup>2</sup>. First, knowledge has a tacit component difficult to replicate, to embody or to codify. However, the share of knowledge that is structurally non-codifiable seems to be small and therefore not so relevant (Cowan *et al.*, 2000) to challenge the Arrow's legacy. Second, the availability and the diffusion of knowledge can be reduced or slowed by appropriation, i.e. measures of knowledge protection and patenting, conceivable as devices to collapse the public good features of knowledge into that of a *latent* public good. Third, knowledge is fragmented and dispersed between a high number of agents, each of them endowed with limited computational capabilities and able to perform mainly local search in the knowledge and technology space (Nelson and Winter, 1982). Fourth and central for this paper, knowledge can be sticky and difficult to exchange or replicate because it is produced to fulfill specific

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<sup>2</sup>I don't tackle here the issue of knowledge definition (Mokyr, 2005; Saviotti, 2007), its externalist or internalist (cognitive) nature, its relation with concepts such as data and information (Boisot and Canals, 2004) or correlated understanding (Metcalf and Ramlogan, 2005) and the forces governing its evolution (Olsson, 2001; Zambelli, 2004; Weitzman, 1998).

purposes. Knowledge is not a homogenous set or structure (Olsson, 2000; Saviotti, 2007), nor it is by definition overlapping across firms and industries; instead, it is directed. As Pavitt puts it: “*most technology is specific, complex. . . [and] cumulative in its development. . . It is specific to firms where most technological activity is carried out, and it is specific to products and processes [ . . . ]*” (Dosi and Nelson, 2010). As a consequence, the exploitation of knowledge externalities becomes a less trivial process, requiring an adequate absorptive capacity (Cohen and Levinthal, 1989) that firms have to actively develop.

An explicit treatment of knowledge is also the most distinctive character of the so-called endogenous and Schumpeterian literature on economic growth (Olsson, 2001). However, this focus on knowledge, probably driven by the need to enquire the very ultimate causes of growth during the worldwide transition from a pure manufacturing-based to a knowledge-based economy, casted shadows more than shed lights: the lack of precision in the definitions of what knowledge is or should be for growth theory, the perfect mapping between knowledge and technology, the puzzling mix between cardinality and ordinality in representing the state, the level or the stock of knowledge, the measurement limits, all led to a “depressing” confusion (Steedman, 2003).

From an analytical point of view, this confusion translates into the use of differential equation to explain the law of motion of knowledge, assuming in most of the cases a linear relation — a knowledge production function — between the growth rate of the knowledge stock and the number of researchers employed in a stylized R&D sector. Subject to knife-edge assumptions and scale-effects (Jones, 1999), a number of refinements to the modeling of knowledge have been suggested in the literature (Dinopoulos and Sener, 2007), unfortunately without modifying the treatment of knowledge as a generic, additive and usually disembodied cardinal stock of resources to accumulate. The analytical advantages to aggregate knowledge in a single scalar are hardly compatible with a picture of heterogeneity emerging from firms’ specific knowledge endowments.

The abstract propositions, prescriptions and procedures borrowed from Science or developed within firms in response to idiosyncratic needs or problems, are realized through, or encoded in, technological artifacts and devices (and also in non-technological ones, such as organizational schemes, routines, institutions, practices). The process of encoding, in turn, guarantees a potentially costless reuse of knowledge (Langlois, 2001), creating the conditions for increasing returns and non-convexities in production to appear. Via the development of technological artifacts or the direct encoding in them, the evolution of knowledge affects differential growth dynamics while remaining in the background. For modeling purposes, this means that knowledge has not to be explicit — as it is instead in endogenous growth models —. Its embodiment in technological components first and then in firms’ capital structure is therefore a realistic modeling choice. Arrow’s learning by doing model (1962b) is the pioneering example in this sense, together with the kaldorian tradition based on technical progress functions (Lorentz and Llerena, 2004), the evolutionary quasi-vintage growth models such as Silverberg and Lehnert (1993) and Silverberg and Verspagen (1994) and the recent reprise of kaldorian arguments in the adaptive economic growth approach (Metcalfe et al., 2006; Foster and Metcalfe, 2010). The present paper aims to give an additional contribution in this trajectory.

### **3 On Problems, Solutions, and Generality**

To adopt an interpretation of knowledge as a firm specific resource that is encoded in technologies and embodied in capital structures constitutes the paper’s first theoretical

building block. Some additional words should now be spent on two related issues: on the one hand, the *determinants* of the production of new knowledge and, on the other, the *characteristics* of this new embodied knowledge. Given that innovative activity is a form of knowledge production, what I will eventually propose is an explanation of the firm's uneven propensity to commit to innovation, a propensity dependent in turn on a process — the emergence of problems and the search for solutions — that is only partially endogenous to the firm.

Concerning the determinants, already in the introduction it was suggested that the need to solve specific problems a firm (and, in general, an economic agent) face is the driver of knowledge production and, accordingly, innovation. Humans and organizations are problem-solvers, making use of procedural rationality (Dosi and Egidi, 1991), heuristics, routines and algorithms to decompose and complete their tasks. Production processes also can be represented — from an engineering perspective — as sequences of task or problems (Kremer, 1993; Buenstorf, 2005; Lombardi, 2009) to be fixed. Also in the evolution of technological systems (Arthur and Polak, 2006), as it happens in human problem solving (Newell *et al.*, 1958), the logic governing progress and learning is that of an iterative succession of duple problem-solution, where every new completed step (a solution found, a new “piece” of knowledge developed) serves as a module or an executable (Arthur, 2009) to be used hierarchically or sequentially for the process to continue.

The need to tackle a precise and defined (sometime ill-defined) bottleneck constraints the range of choices for economic agents and thus imposes a directed (even if not irreversible) structure on the process of change. In sum, and abstracting from other possible determinants, firms become heterogeneous not just because of their initial conditions and expertise (Klepper, 1996), as if they are “born” heterogeneous, but also due to the fact that a unique unfolding process of specific problem-solving activities defines the path and the trajectory of their evolution. Since the development of problem-solving knowledge, exemplified by firms' innovative activities, is actively influenced by the decisions firms take, we cannot treat the problem-solving perspective as a deterministic one. Problems and bottlenecks trace a forced direction, but the choice of certain innovations as solutions between alternatives (combined with the past history of each agent) has the potential to continuously open new opportunities and trajectories. However, at each point in time, a firm capital structure — representing its technological architecture as emerged through time and as a result of sequences of problem-solving activities — tells us the story (at least part of it) of a localized technical change made in response not only to economic incentives opportunities, or inducement by factor prices.

The second issue to deal with is about the properties characterizing the problem-solving knowledge and innovations created and adopted by firms. What makes firms heterogeneous is the fact that the innovations they adopt are heterogeneous themselves. Here it is interesting to measure this dispersion, or variety, in terms of *generality*. Starting with a quote from Herbert Simon, stating that “*no single-purpose device is going to bring about a revolution, however convenient or useful it may be. Revolutionary significance lies in generality*” (Simon, 1987), it is useful to define the concept of generality as the breadth of purposes to which a technology could be applied. In turn, this breadth of purposes is a reflection of the range and types of different problems a technology can solve. A more general technology, embodying a more general problem-solving knowledge, will also have a stronger impact on the technologies complementary to it.

The main idea here is to introduce a “degree of generality”. Each innovation belongs to a continuum of technologies, whose scale is defined by the capability to solve prob-

lems. On the one end of the continuum there are technologies so specific to be able to solve just singular problems, hence pursuing a limited set of functions. On the opposite end one can identify technologies that are very general, introduced in the market for specific reasons but then capable to solve a wider range of problems. Innovations such as the electric dynamo, the laser or semiconductors, initially invented or exploited for specific aims, have later shown to be capable of successfully tackle a much more extensive set of problems.

The recognition that technologies are not all equal is shared with the literature on GPTs (Bresnahan and Trajtenberg, 1995; Helpman, 1998; Carlaw and Lipsey, 2006). GPTs are defined as peculiar technologies that, performing some “generic functions”, display a general applicability, are capable of ongoing technical improvements and foster complementary innovations in application sectors (Rosenberg and Trajtenberg, 2004). The definition and modeling of GPTs suffer from two limitations: the first concerns the ex-ante identification, in the sense that before its diffusion (a process that can last for decades) it is impossible to decide if a technology is general-purpose or not; the second is the problem of classification. Once a technology has been recognized as “important” or revolutionary, it can just fit into one out of two categories: GPT or non-GPT; which should be in this case the empirical criteria to distinguish between what is a GPT and what is not? Given this theoretical shortcomings, GPT-based models (ranging again from industrial economics to growth theory) overcame the puzzle assuming that a GPT is already known to be general-purpose at the moment of its arrival, both in deterministic and in stochastic settings. Except in a case, that of the model of van Zon *et al.* (2003), the general character of a technology never emerge endogenously, because it is already decided from the setup of the models.

The rationale for avoiding the modeling of ex-ante uncertainty about the GPT-nature of a technology is best explained by analytical difficulties. Instead, the decision to invent a new technology — already known to have GPT characteristics — and to invest in it is made in the literature dependent on a double externality, horizontal as well vertical. The latter is based on a dual inducement mechanism, in the sense that the revenue functions of both GPT and application sectors are interdependent: investing more in the GPT increases the returns to invest in complementary technologies, and the other way round. The former type of externality is rather caused by the number of application sectors to which the GPT can be applied. The general applicability part of the GPT definition, often substituted by the weaker formulation “widely used” (Bresnahan and Yin, 2010), refers mostly to the last point: general is meant in terms of diffusion and adoption across a large number of industries and sectors.

The interpretation of generality used in this paper is different with respect to that prevailing in the literature, and it is more coherent with the specification of GPTs as carriers of generic functions (Rosenberg and Trajtenberg, 2004). Moreover, the classification problem is overcome here, since to infinitely different problem-solving capabilities of a technology corresponds an equally infinite range of degrees of generality. Defining generality according to the problems an innovation is able to solve helps us also to understand why, despite the prediction of low-level equilibria and no-investment traps suggested by incentive-based models, important innovations actually occur and diffuse in the economic system.

## 4 Problem-solving and Generality at Work, in a Simple Model

Taking stock of the discussion developed so far, a basic model is now introduced, aimed to show the interaction between firms' problem-solving activities, innovative activities, and performance in terms of productivity and profits. The model is meant to be an initial benchmark, where a single profit-maximizing firm deals with decisions about the R&D investments, which in turn affect both the problem-solving success and the fluctuations in productivity.

The model is a pseudo-vintage model<sup>3</sup> constructed *ad hoc* (but inspired by the arguments of Nelson and Winter (1982), Schuette (1994) and Silverberg and Verspagen (1994)) to focus on the central tenets highlighted so far. The main conceptual difference with respect to the evolutionary literature is that vintages are not competing techniques, subject to selection via a traverse and according to the profits they generate (Nelson and Winter, 1982, Ch. 10). Instead, they are technological *components* of a "compositional" working architecture (Lombardi, 2009). Therefore, even if the firm-level production function analytically permits a certain degree of substitution between the vintages in use, the theoretical production scheme assumed here is that of short-run quasi-fixed proportions model, where complementarities are endemic (Dosi and Grazzi, 2006).

To center the analysis on the structural relation between problem-solving and firm performances, demand is ruled out from the model, so that the market is competitive and always clearing. Also the financial and credit market exists and is perfect; firms can just borrow — at a negligible interest rate — the resources needed to engage in investments. A representative firm  $i$  produces at time  $t$  an output  $q_{i,t}$  equivalent to its revenues with the linear additive production function

$$(1) \quad q_{i,t} = \eta_{it} \sum_{m=1}^M k_{i,m,t}.$$

In what follows, the index  $i$  is dropped to ease the reading. The variable  $\eta_t$  represents the aggregate productivity of the firm,  $k_{m,t}$  is a capital vintage and  $m = 1, \dots, M$  indexes the number of vintages a firm has in use. Concerning labor requirements, in order to keep the model simple, I assume that to each vintage corresponds a worker, so that the number of workers in the firm is equal to  $M_t$ . In each period, the firm incurs a problem that has to be solved. A failure in problem-solving will generate an additional cost  $F$ , whose magnitude can cause the firm to have negative profit and eventually to exit the market. The problem is represented by the absolute value of a real number  $p_t$ , extracted randomly from a Normal distribution with mean zero. I avoid in the model a specification of the qualitative type of the problem<sup>4</sup>. In order to let the nature of the problem to be affected by firm's characteristics, the variance of the distribution is a function of the firm's size, proxied by the number of vintages in use (that corresponds also to the number of workers hired), such that  $\sigma^2 = \left(1 - \frac{1}{M_{t-1}}\right)$ . Larger firms will then have "bigger" problems, as a burden deriving from their complex structure.

The occurrence of problems cannot be avoided, following the realistic argument that

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<sup>3</sup>The use of the word pseudo here is meant to distinguish the use of a generic concept of capital vintage, as in the present model, with respect to the vintage-based growth models in the neo-classical and endogenous tradition, whose principal aim is to determine the optimal scrapping time for different components of the capital stock (Boucekkine et al., 2011).

<sup>4</sup>See for example Cohen *et al.* (1972) for a different approach (the so-called *garbage can* model), aimed to explain organizations' structures and hierarchies more than innovative activities and economic performances.

firms have to continuously deal with bottlenecks and challenges in order to prolong their stay in the market. To impose that, the variance of the distribution must be positive. Therefore, in the model firms enter the market with a minimum of two capital vintages. The emergence of problems introduces strong uncertainty; I assume that the problems distribution is unknown to the agents, preventing them to calculate expected values before the actual arrival of a problem.

Once a problem has emerged, the firm searches for a solution. A solution  $s_t$  can be found or not; the failure in solving the problem decreases the survival probability of the firm. Search is done by investing in R&D, with the aim to develop a specific problem-solving knowledge. As already mentioned, R&D resources can be borrowed on the market without the need to invest in them a share of the profits (which is, instead, the modeling strategy chosen by models such as Nelson and Winter (1982) or Fatas-Villafranca *et al.* (2011)). The main point here concerns the decision about the amount of resources to be invested in research. For a myopic firm, the R&D expenditure will be a function of the expected effect generated on the production process by the new vintage, incorporating the problem solution. In fact, if a solution is found, it is embodied in a new capital component, numbered  $M_t$ . This new vintage is equipped with a specific productivity coefficient  $\eta_{m,t}$ , whose initial value is determined exogenously and it is common for all the vintages (so it can be set equal to one), but whose law of motion will differ across vintages.

The search process for solutions involves again the use of a distribution, this time a Normal centered on the value of the problem ( $\mu = p_t$ ), whose variance is reduced according to the investment in R&D, which are characterized by decreasing returns. Formally,  $\sigma^2 = (\frac{1}{rd_t^\alpha})$ , with  $0 < \alpha < 1$  and  $rd_t$  representing research expenditure. In this sense, R&D expenses can be interpreted as a sort of focusing device, useful to develop a solution specifically directed to solve a problem. By shrinking the distribution around the mean, firms attempt to match the precise value of the problem. While on the one hand it can appear counterintuitive for firms to invest more in absolute terms in order to receive very specific solutions instead of “broader” ones, on the other hand this behavior reduces the risk connected with the failure of solving the problem, that is the occurrence of high additional costs, capable to harm the firm’s production process.

The difference between the value of the problem and that of the solution (conditioned to its finding), so  $s_t - p_t$ , determines the time-invariant *degree of generality* ( $g_{Mt}$ , with  $0 < g_{Mt} < 1$ ) of the solution. Generality is meant here as the amount of problems a certain solution is able to solve. In turn, given that the solution (being specific knowledge) is encoded in a capital-embodied technological innovation, the degree of generality comes to characterize the newly introduced capital vintage. In sum, according to their problem-solving capability, different technological components of the firm’s production architecture will belong to different positions in the generality continuum. A technology characterized by a large positive  $s_t - p_t$  will closely resemble a GPT, while the other way round holds for very specific small technological components. Obviously, a GPT cannot be fully characterized in this model, since the single firm’s behavior is for the moment abstracted from industry and market interactions, excluding that diffusion dynamics take place.

What remains to be defined are the laws of motion for capital and productivity. Each vintage accumulates according to the differential equation  $\dot{k} = -\delta k_{m,t-1} + r_t(g)$ , where  $\delta$  is the rate of capital obsolescence common to all the vintages and  $r_t(g)$  is a parameter which proxies the reconfiguration (or re-switching) that each technological component undergoes because of the arrival of the new vintage. In a very simplified way, the

concept of re-switching (Arthur, 2009) is aimed to capture the architectural connection between the technologies in a given production process: vintages are never scrapped, while the introduction of a new component induces them to a renewal of their structural configuration, so to be “taylor-made” (Aghion and Howitt, 1998) and to fit with the problem-solving innovation just introduced. Therefore, given the arrival of a new vintage, the value of each existing vintage increases in absolute terms if  $r_t(g) > \delta k_{m,t-1}$ .  $r_t(g)$  is assumed to be an increasing function of  $g$ : the more general the new vintage, the bigger the reconfiguration imposed on the existing capital structure.

Firm’s productivity is defined as the capital-share weighted average of each vintage’s productivity

$$(2) \quad \eta_t = \sum_{m=1}^M \left( \eta_{m,t} \left( \frac{k_{m,t}}{K_t} \right) \right)$$

where  $K_t$  is the total capital stock. The law of motion governing the evolution of productivity is  $\dot{\eta} = -c(g) + \eta^g$ , where  $c(g)$  is an initial productivity slump ( $c(g)$  is set equal to zero after the first period, that of introduction in the firm) positively correlated with the degree of generality, and the second term represents the increasing (but marginally decreasing) dynamic contribution to productivity growth, that is smoother when the new vintage has a low degree of generality. As a consequence, a general technology would largely contribute to productivity growth in the medium and long run, but it will inflict a bigger productivity slump in the very short run. The idea here is to impose in a simplified version the so-called “time to sow and time to reap” cyclical dynamics produced by the GPT-based growth models, so the productivity fluctuation due to the lagged effect produced by the adoption of a new GPT. The central mechanism at work in the traditional setting is one in which the adoption of a GPT will generate a productivity slump in the beginning, followed by a major productivity rise. The size and the length of the slump is assumed equal for every cycle, or endogenously determined by the amount of research employed in an intermediate sector producing GPT’s components. Even if the occurrence of slowdowns is not detected for all the technologies, the expectation of fluctuations to happen is quite realistic, and has served as a convincing explanation of the Solow paradox (Solow, 1987; Basu *et al.*, 2007).

The novelty introduced in this model is that fluctuations in productivity arise within each vintage, while the firm’s aggregate productivity is the weighted sum of contribution of all the vintages in use. Therefore, what happens at the firm level results from the interaction of vintage-level states. For example, the initial slump introduced by a very general technology could offset the positive contribution by many specific vintages. Is it better a small slump today or a big rise tomorrow? An “heroic” rational firm should be able to calculate the level of productivity at every period, and to decide how much to invest in research in order to find a problem-solving innovation with a degree of generality capable to exactly contribute to productivity in a measure compatible with time preferences and with the profit maximization problem. However, expectation can be formed only on the value of the solution obtained, while the distribution of the problems is unknown to the agents: firm who can only act as a myopic maximizer.

Speculating on time preferences, firms more apt to prefer higher productivity levels today will invest more in R&D, reducing the risk of failure and also the initial slumps. However, they renounce to bigger gains in the future, as well as to the possibility to discover very general technologies. This fact is in line with what happens in reality, where applied research rarely (or just “serendipitously”) develops revolutionary technologies,

whose general principles and applications are usually discovered in Government-funded or University labs.

Having information only about the current problem, myopic firms try to keep the productivity level high by increasing R&D expenses. Eventually, the firm  $i$  solves its maximization problem

$$(3) \quad \max E(\pi)_t = q_t - (wM_t) - (r(g)_t(M_t - 1)) - F - rd_t,$$

Where  $E(\pi)_t$  are the expected profits at time  $t$  (if  $E(\pi)_t < 0$  the firm exits the market),  $wM_t$  is the wage rate (assumed to be exogenously fixed by mechanisms such as public decisions or collective contracting) multiplied by the number of workers (corresponding to  $M$ , the number of vintages),  $r_t(g)(M_t - 1)$  is the total reconfiguration cost of the existing capital (last vintage excluded),  $F$  is the cost of the problems unsolved, with  $F = 0$  if  $s \geq p$ , and  $F = \epsilon\pi_{t-1}$  (with  $0 < \epsilon < 1$ ) if  $s < p$ ;  $rd_t$  is the investment in Research and Development.

## 5 Conclusions

According to the industrial economics literature, firms engage in innovative activities because of their size or if induced by the structure of the market. Being unsuccessful to confirm empirically the theoretical implications, industrial dynamics scholars introduced new industry-specific explanatory variables in the picture, such as technological opportunities, appropriability conditions, and the characteristics of the industry's technological-knowledge base. In this paper, I suggested that another track can be followed, that of interpreting innovative activities as an outcome of firm's attempt to solve specific problems. While developing solutions to the challenges continuously arising, firms build also their own specific knowledge endowment, which is cumulative and directed. As a consequence, heterogeneity grows fast across the agents.

The results of this problem-solving led innovative activities also influence the degree of turbulence in the market: failing to innovate, firms risks to exit the market. In addition to that, turbulence is affected by the uneven and fluctuating firm's productivity dynamic, in turn depending by the different contribution to output given by each new problem-solving innovation, represented in the model by a vintage of capital. For the last point, the degree of generality of each vintage is a crucial parameter.

Another central assumption of the paper concerns the nature of the firm which, besides being a problem-solving entity, is meant here as a carrier of structural and architectural relations between technological components (and labor).

The basic model proposed in the paper is built on many simplifying assumptions; for example, via their skills workers also embody problem-solving knowledge, and their capabilities can lead to skill-biased technical change, or to particular diffusion patterns (Nelson and Phelps, 1966), but those features are not modeled. Learning processes take place within the firm (Levitt and March, 1988), as well as it happens *by doing* in the production process (Arrow, 1962b). The main motivation of this study is not to deny the traditional sources of incentives for innovative activities (one of which can just be, for example, the capacity of R&D costs spreading for big firms (Cohen, 2010)), but to highlight what seemed to be a missing piece of the puzzle in the neo-Schumpeterian and evolutionary literature about the determinants of innovation.

In conclusion, firms' growth (in term of profits and productivity) and heterogeneity are function of specific and individual capital-vintages-cum-degree-of-generality

mixes; the model can be extended in order to take into account the market interaction between firms, together with the possibility to frame collaborations in innovation (the so-called innovation networks, see Pyka (2002)), imitation and diffusion paths of problem-solving innovations. The behavior of macroeconomic aggregates could then be explained as an emergent property of the micro and meso level *non-simple* (Simon, 1962) interactions and dynamics.

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