Power to the People: The Success of Peer-to-Peer Service

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Abstract
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Abstract

This paper analyses the profitability of a P2P service that competes with traditional retailers. By combining the seminal models by Hotelling (1929) and Salop (1979), we model a cylinder on which products are differentiated along a “local dimension” and a “business dimension”. By choosing a business dimension different to traditional retailers, the P2P service creates a new business. Alternatively, it imitates traditional business concepts. The local dimension stands for any differentiating product characteristic along a horizontal dimension, e.g. geographical location. Our results show that imitation is less beneficial. However, the degree of differentiation along the business dimension crucially depends on the competitive advantage associated with business creation.

Keywords: Product differentiation, imitation, industry structure

JEL-Code: L1, M2
1 Introduction

Founded in 1995, Ebay is among the most successful pioneering peer to peer (P2P) business. There are no intermediate institutions that support the trade of a particular product between buyer and seller, rather P2P services allow for direct exchange between people. They provide a central virtual place where anyone interested can trade the respective good or service via the internet. As a consequence, P2P services compete in the respective market segment with traditional retailers. The field of operation is large. P2P services are not restricted to sales of new or used items (e.g. Ebay) but cover also the exchange of music (e.g. Napster), enable file sharing (e.g. BitTorrent) or facilitate loans between lenders and borrowers (e.g. Zopa). By entering the market, the pioneering P2P service re-shapes the industry structure dramatically. Though the P2P services have the central idea of peer to peer business in common, there is a major difference among them: Some of these companies realise large profits while others fail.

This paper examines when a P2P service competes with traditional retailers by creating a new business. We investigates why some P2P services are more profitable than others. To be precise, we consider implications arising from market entry of a P2P service that is assumed to choose between two strategies: imitation of the retailers’ traditional business concepts or creating a new business. Thereby, the degree of novelty is determined by the P2P service. Crucial determinants for profitability are the intensity of competition to traditional retailers as well as the competitive advantage associated with the novelty of business on the other hand. We discusses strategic implications and the reshaping of industry structure in both cases. An important aspect is that the basic conditions of creating a new P2P service differs from the traditional retailers’ business concept in various aspects, such as costs and profit generation.
To do so, we employ a game-theoretic analysis that combines the two seminal models by Hotelling (1929) and Salop (1979). Products of traditional retailers are differentiated horizontally on a “local dimension” as represented by a Salop circle (Salop, 1979). A P2P service is supposed to enter the market which is inhabited by the traditional retailers. The P2P service either differentiates himself by creating a new business that is choosing a “business dimension” > 0, which is equivalent to accomplishing a different type of business, such as a P2P service in contrast to traditional retailing. Thus, the business dimension represents the way of purchasing: buying online or utilising the service of traditional retailers locally, which is a product attribute along a horizontal attribute space à la Hotelling (1929). Hence, the P2P service’s product differs from the traditional retailers’ products only in the business dimension (if business dimension” > 0). Alternatively, the P2P service can imitate the retailers business with a business dimension equal to 0. The case of new-business creation is equivalent to choosing a position on a cylinder growing out of the Salop circle in our model. This form of competition is less intense and is additionally assumed to comprise a competitive advantage due to the distance to traditional retailers. However, new business creation also entails costs, which are assumed to increase with the degree of differentiation along the business dimension. These costs can be interpreted as the result of development efforts for creating an idea and business plan in the past. Traditional retailers and the P2P service engage in competition for a group of potential customers who decide on purchasing from either of the firms.

In traditional location models, each firm competes with its neighbouring firms as in Hotelling (1929), d’Aspremont, Gabszewicz, and Thisse (1979), Salop (1979) and Eaton and Lipsey (1975). In contrast, our idea of new-business creation is connected to symmetric competition of the P2P service with all traditional retailers in the market. In this respect, our model carries forward the idea of Chamberlin (1933) competition although we use address models. As opposed to our local di-
mension on which traditional retailers are located equidistantly on the Salop circle (with business dimension 0), a clustering effect may occur on the business dimension if the P2P service imitates the retailers business. These kind of clustering effect in attribute spaces have been analysed, e.g. by Shaw (1982), De Palma, Ginsburgh, Papageorgiou, and Thisse (1985) and Swann (1985). In any case, total demand is inelastic in our model and each consumer is supposed to buy one unit of any of the products. Special problems that are associated with inelastic demand and linear transport costs à la Hotelling (1929) can be found in Economides (1984) and Lancaster (1979).

A suite of articles employs differentiation models for very different research questions. In their two-dimensional vertical differentiation model, Vandenbosch and Weinberg (1995) show that if the ranges of attribute spaces are equal among the two characteristics, the firms choose maximum differentiation on one dimension and minimum differentiation on the other dimension. A similar result is obtained by Irmen and Thisse (1998) for a horizontal differentiation model with multiple product characteristics: Firms maximize differentiation in one characteristic but minimize differentiation in the others. There are also articles based on models for products that can be characterised by vertical as well as horizontal product attributes. Given demand uncertainty, Weber (2008) shows that for a monopolist, product versioning in horizontal as well as vertical product dimension is never optimal. Only if the versioning decision can be delayed until demand uncertainty is resolved, there are parameter constellations for which both types of differentiation is beneficial. Lacourbe, Loch, and Kavadias (2009) also consider product positioning of a monopolist. The authors find that the employment of vertical differentiation heavily depend on variable costs. While horizontal differentiation is the main profit lever, vertical differentiation is less beneficial. In response to Weber (2008) and Lacourbe, Loch, and Kavadias (2009), competition is explicitly modeled in our paper but also by various other articles. For example, Economides (1993) analyses consequences of
market entry by modeling a Salop circle plus quality variations. His main finding is an inverse relationship between the quality level and the number of varieties. A similar idea is carried forward by Vogel (2008) who accounts for arbitrarily many firms that are heterogeneous with respect to costs. However, this paper distinguishes itself from the existing literature in that the P2P service has the possibility to create a new product dimension by entering a market of traditional firms. This phenomenon can be observed in reality but, to our knowledge, is not considered in the literature yet.

The remainder of this paper is structured as follows. In section two, we present the game-theoretic model and the business creation choice of the P2P service. The choice of imitation is considered in section three. The fourth section concludes the paper with a discussion of the findings.

2 The Model

The model comprises two types of suppliers: traditional retailer who inhabit the market initially and a P2P service that is supposed to enter the existing market. Supplied products differ in two dimensions, which we will refer to as “local” and “business” dimension. The local product dimension stands for, e.g., the location or facilities of retailers, while the business dimension can be interpreted as the way of purchasing: buying online or utilising the service of traditional retailers locally. With respect to both dimensions, there is no consensus among consumers on the ranking of the products according to these attributes, and thus, both constitute product attributes of horizontal product differentiation.

The local dimension can be interpreted literally as if products of traditional retailers are supplied at different geographic locations. A more general interpretation
is that products differ in one characteristic, e.g. the carbon dioxide content of mineral water. One can even say that the local dimension covers different kind of goods, such as clothes, shoes and accessories. This is true as long as people have different tastes and do not agree on ranking the products with respect to their quality. The P2P service’s product differs from the traditional retailers’ products only in the business dimension (if business dimension” > 0), which can be interpreted twofold. One can argue that offering products online or providing a marketplace for buying and selling does not require local establishments and therefore, the P2P service competes with all locally established retailers equally. If we interpret the local dimension less literally, one can say that as P2P trade does not require storage rooms or equipments so that the P2P service is not necessarily specialized like traditional retailers. E.g. Ebay does not confine itself to a particular product while local competitors, e.g. fashion houses do. Thus Ebay competes with The Home Depot for home improvement and construction products and services but also with Walgreens for drugs and healthcare products.

Figure 1 illustrates the market structure. Products of traditional suppliers can be assigned to a point on a circle (Salop, 1979) with a perimeter equal to 1 according to the local product dimension. \( n = 4 \) traditional Retailers are equidistantly positioned on the circle at the bottom of the cylinder. In contrast, the P2P service can be represented by the blue area at the top of the cylinder. In Figure 1, the P2P service is located at maximum business dimension equal to one. Consumers have different tastes with respect to the product attributes and are uniformly distributed on the cylinder along the local dimension \([0,1]\) so that they cover the surface of the cylinder in business dimension on the interval \([0,1]\). Each consumer buys one of the products. If the consumer has to buy a product that does not match his preferences perfectly, he pays transportation costs, which arise from the compromise of buying a less-than-ideal product. These transportation cost are linear as in the pioneering work by Hotelling (1929) and Salop (1979). For example, a particular consumer
who is located at position $x$ in Figure 1, faces transportation cost according to the distance $x \overline{a}$ on the local dimension and $\overline{a} i$ on the business dimension if he buys from traditional retailer $i$. If he decides to buy from the P2P service, he faces costs according to distance $\overline{bx}$.

The timing of the game is as follows. The P2P service enters the market that already comprises a fixed number of traditional retailers in the first stage of the game by choosing its positioning on the business dimension. In the second stage of the game, all firms simultaneously compete in prices. Afterwards, each consumer buys exactly one unit of the preferred product.

To be more precise, consumer $x$’s indirect utility function is given by

$$ u_{x,i} = \bar{u} - p_i - t(x_i - 0) - t(x_b - 0) \quad (1) $$

$$ u_{x,j} = \bar{u} - p_j - t \left( \frac{1}{n} - x_j \right) - t(x_b - 0) \quad (2) $$

if he buys from firm $i$ ($j$) whose local position as well as its business position is
normalised to zero $y_{i,l} = 0$ and $y_{i,b} = 0$ ($y_{j,l} = \frac{1}{n}$ and $y_{j,b} = 0$) according to our assumptions. $x_l$ and $x_b$ denote consumer $x$’s coordinates in local and business dimensions and $t$ denotes the cost parameter for transportation in either business dimension or local dimension. Reservation utility $\bar{u}$ is assumed to be sufficiently high so that $x$ buys exactly one unit of either product. $p_i$ ($p_j$) as well as $p_s$ used below denote prices of traditional retailer $i$ ($j$) and the P2P service $s$, respectively. The consumer’s utility from buying from the P2P service instead is given by

$$u_{x,s} = \bar{u} - p_s - t(b - x_b).$$ (3)

In the local dimension, there is an indifferent consumer whose utility from buying product $i$ and buying product $j$ is equal for business dimension $x_b = 0$. His position $\hat{x}_{ij}$ can be obtained by equating utility functions (1) and (2) which yields

$$\hat{x}_{ij} = \frac{np_h - np_i + t}{2nt}. \quad (4)$$

Similarly, we obtain position $\hat{x}_{hi}$ of the indifferent consumer whose utility from buying product $h$ is equal to his utility when buying product $i$

$$\hat{x}_{hi} = \frac{np_i - np_h - t}{2nt}. \quad (5)$$

Due to symmetry, all consumers with business location $> 0$ that are indifferent between buying from either of the retailers have the same position on local dimension as given by $\hat{x}_{hi}$, see the red-dotted vertical lines in Figure 2.

Figure 2 is the front view of the cylinder and encapsulates the calculation of consumer demand. In order to do so, we have to find the consumer $\hat{x}_{is}$ who receives the same utility from buying from retailer $i$ or buying from the P2P service to yield the upper bound of the blue pentagon in Figure 2, that incorporates retailer $i$’s demand. $\hat{x}_{is}$ can be found analogously to the calculations above.

$$\hat{x}_{is} = \begin{cases} 
\frac{p_s - p_i + tb + tx_l}{2t} & \text{if } x_l < 0 \\
\frac{p_s - p_i + tb - tx_l}{2t} & \text{if } x_l > 0 
\end{cases} \quad (6)$$
If $0 < \hat{x}_{is} < 1$, $\hat{x}_{is}$ constitutes the indifferent consumer in the business dimension. Equation (6) corresponds to the blue-dotted line in Figure 2, which is symmetric with respect to retailer $i$’s local position if neighbouring firms set equal prices.

From this, we can calculate retailer $i$’s demand $q_i$ as the sum of the area left and right of $i$’s local position

$$q_i = \left| \int_{\hat{x}_{hi}}^{0} \hat{x}_{is} dx_l \right| + \int_{0}^{\hat{x}_{ij}} \hat{x}_{is} dx_l$$

$$= \frac{(n(p_h - p_i) + t)(n(4(p_s + tb) - p_h - 3p_i) - t)}{16n^2t^2}$$

$$+ \frac{(n(p_i - p_j) - t)(t + n(3p_i + p_j - 4(p_s + tb)))}{16n^2t^2}$$

As retailers are located on the circle equidistantly, total retailer demand equals $nq_i$. Demand of the P2P service is given by the white area in Figure 2 covering those consumers whose preferences in business location are in between the maximum 1 and the indifferent consumer $\hat{x}_{is}$, $q_s = 1 - nq_i$.

All firms maximise profits, which are given for retailer $i$ and the P2P service
by

\[ \pi_i = p_i q_i - F_R \]  \hspace{1cm} (8)  
\[ \pi_s = p_s q_s - K_s. \]  \hspace{1cm} (9)

The costs of the P2P service \( K_s := F_s + b^2 - \phi b \) incorporate the trade-off related to the positioning on the business dimension: The first term \( F_s \) are fixed cost, the second term \( b^2 \) can be interpreted as the costs of development efforts for creating an idea and business plan in the past. The third term reduces costs by granting a special advantage for choosing a position far away from local retailers and serving consumers at the higher end of the business scale. \( \phi \), with \( 0 < \phi < b \), is a scaling parameter that determines the impact of the P2P service’s competitive advantage.

\( F_R \) stands for costs of traditional retailers that are independent of price choices and are equal among them. In the traditional Salop model (Salop, 1979), these costs are crucial as they determine the number of firms competing on the circle. In this model, firms enter the market as long as there are positive profits. Thus, equating overall profits to zero, yields the number of firms \( \frac{\sqrt{t}}{\sqrt{F_R}} \). Our perspective is different, as we are interested in the development and chances of new business models in traditional structures. We therefore explicitly model an attractive environment where profit generation is possible. Other factors determine profits and keep the number of retailers low, such as entry barriers, which are not internalised in our model. This corresponds for \( n = 4 \) retailers to \( F_R \leq \frac{t}{16} \) and in the absence of the P2P service, retailer gain positive profits if \( F_R < \frac{t}{16} \) holds.

### 2.1 Pricing Subgame

On the last stage of the game, the four traditional retailers and the P2P service compete for consumers by maximising profits (8) and (9) which yields best responses
for optimal prices
\[
p_i(p_s) = \frac{1}{9}(p_h + p_j + 4(p_s + tb)) + \frac{1}{18}t - \frac{1}{36} \cdot \sqrt{88 \cdot (p_h^2 + p_j^2) + 256p_s^2 + 4p_h(8p_j - 40p_s)} \tag{10}
\]
\[
p_s(p_i) = \frac{8(p_h^2 + p_j^2 - 6p_i^2) + 16p_i(p_h + p_j) + 4t(p_h + p_j + 2p_i)}{64(p_h + p_j - 2p_i) + 32t} - \frac{16tb(2p_h + 2p_j - 4p_i + t) + 33t^2}{64(p_h + p_j - 2p_i) + 32t} \tag{11}
\]

At this stage, we make use of our assumption that traditional retailers are symmetric with respect to costs. In this case of symmetry, retailers will choose equal prices, e.g. \( p_j = p_i \). Optimal prices then read
\[
p_i = \frac{1}{32} \left( 41t + 16tb - \sqrt{1185t^2 + 1056t^2b + 256t^2b^2} \right) \tag{12}
\]
\[
p_s = \frac{1}{64} \left( 107t - 16tb - \sqrt{1185t^2 + 1056t^2b + 256t^2b^2} \right). \tag{13}
\]

Competition on the ground Salop circle is higher compared to competition between retailers and the P2P service. This becomes apparent here, as the P2P service’s price (13) is always larger than the traditional retailers’ prices, such as \( p_s > p_i \) holds for \( t > 0 \) and \( 0 \leq b \leq 1 \). The same holds true for demand \( q_s > q_i \).

Profits including optimal prices read
\[
\pi_i = \frac{1}{4096} \left( 5 \sqrt{t^2(1185 + 1056b + 256b^2)} - t(81 + 16b) \right) - F_R \tag{14}
\]
\[
\pi_s = \frac{t}{4096} \left( 6317 + 32b(-37 + 8b) \right) - \frac{107}{4096} \sqrt{t^2(1185 + 32b(33 + 8b))}
+ \frac{b}{256} \left( -256b + \sqrt{t^2(1185 + 32b(33 + 8b))} \right) + b\phi - F_s. \tag{15}
\]

Fixed costs \( F_R \) are required to be not too high\(^1 \) to ensure non-negative retailer profits. The P2P service’s profits are not necessarily greater than the retailer ones. For \( F_R < F_s \), even the opposite may hold true \( \pi_i > \pi_s \), see Appendix 5.1 for details.

\(^1\) \( F_R \leq \frac{81}{4096} t - \frac{1}{256}tb + \frac{5}{4096} \sqrt{1185t^2 + 1056t^2b + 256t^2b^2} \)
2.2 Business Location Subgame

The P2P service maximises profits (15) with respect to location \( b \) on the business dimension which yields first order condition

\[
\frac{\partial \pi_s}{\partial b} = \frac{1}{128} t(16b - 37) + \frac{t^2(-1173 + 64b(4b - 1))}{128 \sqrt{t^2(1185 + 32b(33 + 8b))}} + \phi - 2b = 0 \tag{16}
\]

that determines the optimal business location if \( 0 \leq b \leq 1 \). If this condition is not met, the P2P service will locate at either the bottom of the cylinder \( b = 0 \) or at the top \( b = 1 \). Respective corner profits are a simplification of equation (15) and read

\[
\pi_s(b = 0) = \frac{6317t - 107\sqrt{1185}\sqrt{t^2}}{4096} - F_s
\]

\[
\pi_s(b = 1) = \frac{5389t - 91\sqrt{2497}\sqrt{t^2}}{4096} + \phi - 1 - F_s.
\]

If there is no interior solution to the business dimension decision, only the relationship of \( \phi \) and the transportation cost parameter \( t \) matters. If the competitive advantage \( \phi > 1 \) is over-proportional\(^2\) and satisfies

\[
\phi > 1 + 0.43748t, \tag{17}
\]

the P2P service will always prefer the business position farthest away from the traditional retailers as \( \pi_s(b = 1) > \pi_s(b = 0) \). However, since the business dimension is given by the interval \( b \in [0, 1] \) and \( \phi \) is restricted to \( \phi < b \), this condition will never hold. Thus, the P2P service will always favour \( b = 0 \) over \( b = 1 \). This is true because the P2P service ensures demand (which is called the backyard) at the top of the cylinder by choosing the minimum business dimension, although price competition is more intense. In the following, we analyse the P2P service’s positioning decision along the business dimension for first, \( \phi = 0 \) that implies the absence of a competitive advantage. Second, we look at the more general case, where \( \phi > 0 \) specifies the level of the competitive advantage.

\(^2\)Rearranging equation (17) to make \( t \) the subject, \( t < -2.28582 + 2.28582\phi \), shows that as \( t > 0, \phi \) must satisfy \( \phi > 1 \).
In the Absence of Competitive Advantage

If there is no competitive advantage, $\phi = 0$, of positioning along the business dimension, then the P2P service will locate at the bottom of the cylinder, with business location equal to zero. Here, $\frac{\partial \pi}{\partial b} < 0$ as all three expressions in derivative (16) are negative for $\phi = 0$ and $0 \leq b \leq 1$. In this case, position-related costs determine the outcome. Demand is equally distributed among traditional retailers and the P2P service, $nq_i = q_s$.

In contrast, a position $b > 0$ on the business scale would cause price competition to get less fierce. However, this effect is outweighed by the opposed effect of positioning-related costs. In the end, the P2P service immediate competes with traditional retailers. Referring to our findings above, we find that profits of a traditional retailer exceed the P2P service’s costs if respective fixed costs $F_R$ but also transportation costs determined by $t$ are low enough, $0 < F_R < F_s$ and $0 < t < \frac{2048}{7(8\sqrt{1185} - 457)}(F_R - F_s)$.

In the Presence of Competitive Advantage

In the presence of competitive advantage, $\phi > 0$, we hold on to the finding that maximum location will never be beneficial. To make the analysis tractable, we restrict our analysis to a graphical solution for $0 < t < 2$, $0 \leq \phi < 1$ and $F_o = 0$. In Graph 4, the blue-transparent plane is the profit in case of an interior solution $0 < b < 1$. The green striped plane shows that profits in case of $b = 0$ are greater than those provided by the interior solution for some $(\phi, t)$-combinations. If the competitive advantage is under-proportional, $\phi < 1$, the P2P service will locate at the bottom of the cylinder for small values of the transportation cost parameter $t$. However, if transportation costs are close to the maximum, $t < 2$, interior profits are larger. This is due to the fact that high transportation costs imply another
advantage for the P2P service. Consumers who bear the transportation costs, face these cost in horizontal as well as vertical direction if they buy from a traditional retailer. As the P2P service covers all local dimension instead, consumers face utility losses only along the vertical distance in business dimension. In other words, the larger \( t \), the lower is the position of the consumer who is indifferent between buying from a traditional retailer and the P2P service \( \bar{x}_{is} \) and the lower the retailers’ demand.

### 3 Imitation of the Tried and Trusted

The P2P service might enter the market by creating a new business and choosing a location along the business dimension \( b \geq 0 \) as considered above. However, the alternative is to imitate the business concept of traditional retailer and settle somewhere on the Salop circle with \( b = 0 \) so that the number of retailers that are distributed equidistantly on the circle increases to \( n = 5 \). As we assumed that the retailers’ fixed costs \( F_R \) are sufficiently low, market entry is still beneficial for \( b = 0 \).
Computations are standard and taken from Salop (1979). In the Salop case, profits of traditional retailer $i$ are given by

$$\pi_i^S = \frac{p_i(n(p_j - p_i) + t)}{nt}$$

(18)

in the absence of costs and for $p_h = p_j$. Optimisation with respect to prices yields

$$p_i^S = \frac{np_j + t}{2n}$$

(19)

and simplifies for symmetric prices $p_j = p_i$ and $n = 5$ retailers to $p_i = \frac{t}{5}$. Each retailer generates profits that amount to $\pi_i^S = \frac{t}{n^2} = \frac{t}{25}$. Adding these profits into Graph 4 by a pink meshed plane (and omitting corner profits $b = 1$), yields the following picture which shows that the P2P service’s profits are always less in case of imitating existing retailers. The case of imitation requires that the P2P service shares demand equally with competitors as he covers a bounded area on the Salop circle. In contrast, creating a new business model incorporates the advantage that demand is shared equally among all retailers and the P2P service and is therefore preferable. However, the existence of fixed cost might change the situation completely but is not considered here.

Figure 4: Profits of the P2P Service including Imitation
4 Conclusion

This paper analyzes the profitability of a P2P service who competes with traditional retailers. By combining the seminal models by Hotelling (1929) and Salop (1979), we model a cylinder on which products are differentiated along a “local dimension” and a “business dimension”. The P2P service is supposed to create a new business model by choosing the business dimension or alternatively to imitate traditional business concepts. Our results show that imitation is less beneficial. However, the degree of differentiation along the business dimension crucially depends on the competitive advantage associated with the business creation.
References


5 Appendix

5.1 Pricing Game - Profits

The retailers last stage profits (14) exceed the P2P service’s profits (15) if either

\[
b = 0 \land 0 < F_R < F_s
\]

\[
\land 0 < t \leq \frac{2048(-3199F_R+3199F_s)}{6517441} + \frac{16384\sqrt{58065F^2_R-116130F_RF_s+58065F^2_s}}{6517441}
\]

or the following holds

\[
0 < b \leq 1 \land 0 < F_R < F_s + b^2 - b\phi
\]

\[
\land \phi < \frac{F_s+b^2}{b}
\]

\[
\land 0 < t \leq 2048 \left( \frac{(3199+86(-73+16b))(-F_R+F_s+b(b-b\phi))}{6517441+16b(-374143+166(4795+48b))} \right) + 8 \sqrt{\frac{(1-7+b)^2(1185+32b(33+8b))(-F_R+F_s+b(b-b\phi))^2}{(6517441+16b(-374143+166(4795+48b))^2)}}
\].