A Counterfactual Analysis of the European Unitary Patent

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Abstract

Year of enrolment: 2018  
Expected final date: summer 2021  
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December 3, 2019

Abstract

We analyze how the harmonization of patent systems affects patent value and patenting behavior in Europe. We model the validation and renewal decisions for European patents granted by the European Patent Office to companies during the period 1998-2003. Our model allows us to assess the private economic value of European patents including the correlation between returns across countries. In a second step, we perform a counterfactual analysis of the Unitary Patent system. This new European patent regime intends to simplify and reduce the cost of patenting in Europe. Our goal is to quantify some of the benefits and costs of this new regime.

Please, note that this project is still at a very early stage. This draft paper shows the method we wish to adopt to address our research question.

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*Financial support from Hanken School of Economics and the IPR University Center is gratefully acknowledged. We would like to thank Mattia Nardotto and participants at the KU Leuven IO seminar for comments and Adrian Kovalic for advise on the PATSTAT data. All errors our ours.

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1 Introduction

We analyze the effects of introducing the European patent with unitary effect (the Unitary Patent (UP) for short). Over the past 50 years the European patent systems have evolved with increased integration and harmonization.\footnote{Pila (2013) provides a full history of the different proposals for patent reform in Europe} The European Patent Convention (EPC) in 1973 created the European Patent Office (EPO) which provides a legal framework for the granting of European patents (EPs). An EP has a single harmonized application and granting procedure but remains essentially a bundle of national patents which offer protection for the same invention in multiple EPC Member States. The implementation of a real European patent with a centralized post-grant procedure has remained elusive until recently. The new UP system is finally expected to be introduced in the first semester of 2020, allowing for a single harmonized post-grant procedure that will significantly reduce the costs of patenting in Europe.\footnote{The introduction of the UP is linked to the creation of the Unified Patent Court (UPC) which will have jurisdiction over UPs and EPs. All the participating members have to ratify the UPC Agreement Act. At the time of the writing, the ratification process is on hold after a constitutional complaint in Germany.}

To evaluate the expected effects of the UP system, we first build and estimate a structural model of validation and renewal decisions for EPs granted to applicants in five technological areas between 1998 and 2003. We then use the estimates to analyse the counterfactual effects of the UP on the private value of patents as well as the social value. In the UP scenario, the annual renewal fees are unique and the UP covers \textit{de facto} the full set of Member States. Given that the patentees will be given the option to apply for a UP instead of a country-specific EPs, the introduction of the UP can only increase the value of patents. The new system will reduce significantly the cost of patenting. The territorial scope of the patent increases because the UP covers all States signing the UPC Agreement. Both the change in fees and the change in territorial scope will affect the privately optimal length of the patent. At the extensive margin, new inventions that would not be applied for protection or validated under the current system will be in the new system. These
changes will not only affect the patents that would be applied for under the current EP, but could also increase incentives to invent. At the same time, by making patent protection more cost effective, the UP will enhance propensity to patent and may hence worsen the functioning of the patent system by increasing the number of patents per R&D euro.

Our model captures the different effects of the UP: territorial scope, length, cost-reduction and measure them under the UP scenario. In a next version of the paper, we will try to estimate the effect in terms of new inventions (extensive margin effect). These effects also depend on the interactions between returns in different countries that we are modelling following the approach of Deng (2011). At the time of writing, we find some preliminary results of correlation between returns across countries and we provide a preliminary counterfactual analysis. Note that these results are very likely to change.

There are numerous studies of the effects of the design of patent systems on the rate of invention. Most of the empirical work uses changes in patent policy as natural experiments to tackle this question (e.g. Sakakibara and Branstetter, 1999; Lanjouw and Cockburn, 2001; Lerner, 2002; Moser, 2005; Qian, 2007). A few papers look specifically at the implications of patent protection harmonization. Studies in this literature mostly focus on implications of strengthening patent protection for the less innovative South versus the more innovative North (e.g Helpman, 1993; Lai and Qiu, 2003; McCalmont, 2001; Bilir, Moser and Talis, 2011). Moreover, the existing literature is mostly about ex-post policy evaluation whereas we develop a tool for ex-ante policy evaluation. Closer to our work, Hall and Helmers (2019) analyze the patenting behavior of firms following the accession of 14 countries to the EPC in the last decade. Also, Danguy and de la Potterie (2011) estimate the likely impact of the UP on renewal fees and costs of patenting. We contribute to this literature by building a structural model of validation and renewal decisions of EPs. The model allows us to estimate the private value of EPs up to 15 countries. We then use our estimates and our model to conduct a counterfactual analysis of the effects of the UP.

Our model borrows from the literature on patent renewal initiated by Pakes and
Schankerman (1979, 1984). They develop a deterministic model where the renewal decision is an optimizing process balancing the renewal costs with the expected returns of generated by the patent. Every period, the patent holder has to decide whether or not to pay a renewal fee to keep the patent in force. Based on these decisions, it is possible to infer a distribution for private value of patents. This framework has been extended in different directions: to take into account learning effects and stochastic shocks (Pakes, 1986), litigation costs (Lanjouw, 1998), trade of patents (Serrano, 2018) and the international framework (Putnam, 1997; Deng, 2007, 2011). At this stage of the project, we estimate a deterministic model in two stages. First, we estimate on country-specific ordered probits with the computation of generalized residuals following Gourieroux et al. (1987). It allows us to recover estimates for correlations between returns in different countries. Considering the large number of countries, this method has a great computational advantage and provides good starting values to our next stage where we use the composite marginal likelihood (Varin, Reid and Firth, 2011, for an overview) method. In a next step, we plan to adopt a stochastic framework similar to Deng (2011).

Our empirical work should help policy makers on different levels. First, it will give insight to better understand the effects on patenting behavior of the harmonization of the European Patent System, through a centralized post-grant procedure. Second, it provides a basic model to test the impact of different fee schemes and evaluate how they affect the private value of patent through scope effects and length effects. In this regard, this work is related to papers looking at the effects of fees on patenting behavior (De Rassenfosse and van Pottelsbergh de la Potterie, 2013, for a survey). Third, it we extend the existing literature by providing estimates of the synergies for inventions protected in multiple countries through the correlation between returns of patents in different countries.

The rest of the paper is organized as follows. Section 2 presents the European Patent System and the distinction between EP and UP. Section 3 describes the data source and provides some descriptive statistics. Section 4 introduces the theoretical framework. Sec-
tion 5 presents some preliminary results for both estimation and our counterfactual analysis. Section 6 concludes.

2 Institutional background

The European Patent System includes three layers: national patents, EPs and the upcoming UPs. In this paper, we focus on EPs and UPs.

2.1 European Patents

Since 1977, the European Patent Office (EPO) offers a unified patent application and examination procedure for all of the signatory States to the European Patent Convention (EPC). In 1978, only seven members were contracting States and 3,572 patents were filed. Today 38 countries are contracting States\(^3\) and 145,394 applications were filed in 2016. The terminology ”European” is misleading because the European dimension is only at the examination of the patent application. In fact, EP is not a supranational patent but a group of national patents. Therefore, an EP is subject to national patent law, including the payment of renewal fees in States where the patent is in force. This fragmented and complex post-grant procedure results in a more expensive patent system than in the US or in Japan (van Pottelsberghe de la Potterie andFrançois, 2009). It is one of the arguments raised in the long-lasting debate as to why Europe should introduce a harmonized patent system.

Application and Examination. In the first stage, the applicant files an application for an EP in one of the three official languages (English, French or German). At the time

of the application, the applicant pays a standard filing cost including a European search fee and an examination fee. Within twelve months after the filing date, the applicant is free to choose the Member States in which to seek for protection and pays per-country designation fees. Since 1999, all countries are designated by default, so most applicants decide to designate the full set of EPC Member States. Moreover, the designation fee scheme encourages applicants to seek protection in the full set of States. In fact, per-country designation fees are identical for each country, up to a maximum of seven, after which it becomes free of charge. The period of examination lasts for two to six years before the grant decision for the patent is given. During this period of examination, the EPO conducts a formality check and then a search report describing the state of prior art. The patent examiners evaluate if the EPO requirements for patentability are met, which are novelty, inventive step and industrial application. The search report and the application are published in the EPO Bulletin 18 months after the priority date of the patent application. The applicant may request the examination within six months after the publication of the application. Not requesting the examination is equivalent to withdrawing the patent.

Validation and Renewal decisions. After the period of examination, the patent is approved or denied by the EPO. Traditionally, the EPO grants 60-65% of the patent applications, refuses 5%, and 30-35% are withdrawn by the applicant during the search and examination process (Lazaridis and de la Potterie, 2007). If the patent is granted, the assignee decides whether to pay an additional lump-sum cost (translation and other administrative costs for extension/validation) in each designated country to be able to validate and then transfer the granted patent into national laws. In the following, we will call these costs validation costs. In practice, applicants do not validate in all the designated states. The validation costs differ between countries and patent characteristics. For instance, some translations notably in the languages spoken in Nordic countries (Danish,

\footnote{In reality, some applicants decide to opt-out from some States for litigation reasons. In the first approximation, we ignore these cases.}
Swedish, Finnish) are more expensive. The translation service is usually provided by the local attorney and depends on the size of the patent (number of pages) and the patent characteristics. Since the London Agreement was signed in 2008, translation costs have decreased. In fact, signatory countries to the London Agreement do not require that the applicants obtain a full translation of the patent into the local language, only the claims of the EP are required. Moreover, some States do not require translation at all (Germany, France, United Kingdom, Austria, Switzerland, Belgium, Ireland, Monaco or Luxembourg). According to Harhoff et al. (2009), translations are not required in 60% of validation cases. Regarding the other administrative costs of validation, they also differ by country. Some countries charge a fixed fee; some countries do not charge a fee (Belgium, Switzerland, Luxembourg, Monaco and the UK), some countries charge an additional page-based fee for patents exceeding a certain size (Austria, Finland, Sweden, Denmark and Spain) (Harhoff et al., 2009). In fact, per-country validation costs are very difficult to measure. Therefore, we have decided to estimate them in our model of validation and renewal decisions.

Once the patent is validated in a country, it gives the same right as the national patent and it is valid for up to 20 years from the filing date. National patent laws apply thereafter, including the requirement to pay an annual patent renewal fee in order to keep the patent alive. The fee scheme is fixed and varies across countries (see appendix A.1 for renewal fees for the patent granted in 1998-2003). Harhoff et al. (2009) shows that the level of renewal fees, validation costs as well as translation costs have an impact on the choice of countries chosen for validation. Our model includes this trade-off in the choice of validation countries.

Figure 1 is a simplified presentation of the patent lifetime (timeline is indicative). In practice different routes exist such as first filings, second filings and PCT applications. Guellec and Van Pottelsbergh de la Potterie (2007) provides an in-depth description of filing procedures at the EPO. Our model focuses on validation and renewal decisions.
2.2 The Unitary Patent

**Principle.** The Unitary Patent System is expected to start in the first semester of 2020 and will become an additional option alongside the current national patents and EPs. As mentioned previously, it has been discussed in one form or another for more than four decades. The centralized pre-grant phase will remain the same under the new regime. In other words, there won’t be any difference in the quality of search and examination conducted by the EPO for EPs and UPs. The difference with the EP will be in the post-grant procedure with the introduction of a unique procedure, currency, deadline and no obligation to use a representative. Once the EP is granted, the applicant will be allowed to ”request for unitary effect” at the EPO. This request will be free of charge and must be filed in the month following the publication of the grant in the European Patent Bulletin. Moreover, a condition to be eligible for the unitary effect is that the EP has to be granted in at least the same set of States covered by the UP system. After the request, the EP will become a UP. Whereas the EP is validated and renewed separately in each State, the UP will be renewed once a year in the full set of States covered by the new regime.

**Scope of the Unitary Patent.** The UP will give a protection in up to 26 EU Member States which have ratified the UPC agreement. According to the EPO, it is very likely that other EU countries will join the Unitary Patent System in the following years. The territorial scope of UP is then likely to increase but the UP will have a fixed coverage based on the date of registration of the patent. In other words, multiple generations of UP with

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### Figure 1: Life-cycle and costs of a European Patent

<table>
<thead>
<tr>
<th>Application</th>
<th>Granting decision</th>
<th>Renewal decision</th>
<th>End of protection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designation fee in each country</td>
<td>Validation costs in each country</td>
<td>Renewal fee paid in each country</td>
<td>Last renewal fee</td>
</tr>
<tr>
<td>Search and examination fees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EPO renewal fees starting in 3rd year</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>year 0</th>
<th>year 4</th>
<th>year 5</th>
<th>year 20</th>
</tr>
</thead>
</table>

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different coverage are expected to be in force at the same time. This point is important as different combinations of EP and UP will be possible. Even with a Unitary Patent, it will still be necessary to go through validation or extension in EPC states that are not in the UP system. In our model, for simplification we look at EPs granted between 1998-2003 in 15 countries and our counterfactual policy focuses on the full set of countries. We then rule out the possibility of a combination between EP and UP in our model, as well as a combination between national patents and European Patents. A possible consequence of the co-existence of these systems is to see a substantial number of duplicate patent filings across the different systems. von Graevenitz and Garanasvili (2018) demonstrate the existence of double patents at the national and European level.

Costs of UP. The UP system will significantly decrease the cost of patenting compared to the EP because the validation costs will be suppressed and the renewal fees will be unique and lower. The renewal fees for the UP are already known (see annex A.2). Unique renewal fees would have to be paid the years following the year in which the UP has been granted. Similarly to the EP, if renewal fees are not paid on time, the UP will lapse. Nevertheless, it is still possible to pay within six months of the due date with a penalty of 50% of the belated renewal fee. The renewal fee scheme is set to be equal to the sum of EP renewal fees in the four most popular countries in 2015 for EP patent protection (Germany, France, the UK and the Netherlands). According to the EPO, it costs 170,000 euros to obtain a EP for ten years in 25 states covered by the UP, whereas it will cost only approximately 35,000 euros with a UP. In the long run, a UP will not require the translation of the patent which is one reason for the high cost of EP (van Pottelsberghe de la Potterie and François, 2009). Nevertheless, in a six-year transitional period (which might be extended to up to 12 years), translation will still be required. The UP, similarly to the EP, will have to be filed in English, French or German, the so-called procedural languages. Patents in English will need to be translated into one of the other procedural languages. French and German language patents will be translated in English. To compensate applicants for the added
cost during the transitional period, the EPO will launch a scheme to cover costs related to
the translation of the patent application for EU-based SMEs, natural persons, non-profit
organizations and universities that are resident in a contracting Member State.

Unified Patent Court. The new regime is linked to the creation of the Unified Patent
Court. The UPC will have jurisdiction on both EP and UP. The creation of the UPC will
have effects on litigation costs as it will provide a unified court to centralize litigation. We
can expect that the creation of the Court will have an effect on the value and incentives
to innovate. In this version of the paper, the effects of litigation are neglected. Also the
Unitary Patent can be licensed in respect of the whole or part of the territories of the EU
Member States. It is likely that some patent owners will take into account the future cost
of litigation when they will decide which route (national, EP, UP) to choose. The fact that
the Unitary Patent can be revoked in a single action in respect of all participating members
may also reduce the appeal in respect of high-value patents. In contrast, a standard EP
could only be revoked on a national basis, one state at a time.

Figure 2 is a simplified presentation of the patent lifetime under the UP

Figure 2: Life-cycle and costs of a Unitary Patent

<table>
<thead>
<tr>
<th>Application</th>
<th>Granting decision</th>
<th>Renewal decision</th>
<th>End of protection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designation fee in all states</td>
<td>Request for unitary effects (free)</td>
<td>Single renewal fee for all countries</td>
<td>Last single renewal fee</td>
</tr>
<tr>
<td>Search and examination fees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EPO renewal fees starting in 3rd year</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3 Data

3.1 Data source

The patent data comes from the EPO PATSTAT database (spring 2019) and records all
EP applications. It includes information on the designation (at the time of the applica-
tion) and validation (at the time of the grant) decisions as well as the full renewal history. PATSTAT also provides patent characteristics that are relevant to proxy for the value of returns: Number of forward citations, number of inventors/applicants, number of claims, IPC classes. Our sample consists of all the EPs issued between 1998 and 2003, and designated in the set of 15 Member States. There are 61,968 patents in our sample. We choose to focus on patents granted between 1998 and 2003 for two reasons. First, we are able to observe the full life for these patents: Up to 20 years after patent filing which is 16 years after the patent grant decision if the examination period is 4 years. Second, the number of Member States of the EPC during this period is quite stable. In fact, as the examination period lasts for an average of 4 years, most of the patents in our data have been filed between 1994 and 1999. During this period, only two new states joined the Convention: Finland (1996) and Cyprus (1998). In the next version of the paper we will use more recent data. The model will then include more States, as 18 more countries joined the Convention between 2000 and 2010. In this context we will adapt our model to take into account censoring because we will not observe the full life of patents.

Renewal fees for each country are extracted from the EPO’s reports: "National Law relating to the EPC" for each relevant year. Fees are expressed in euro 2000 and reported in Table 4. Note that there exists other costs for the EP such as representation costs (attorney fees, other service providers), translation costs incurred for validation and/or publication. In the model, we assume only two type of costs: The renewal fees and the validation costs paid the first period following the grant of the patent.

5Austria (AT), Belgium (BE), Switzerland (CH), Germany (DE), Denmark (DK), Spain (ES), France (FR), Great Britain (GB), Greece (GR), Ireland (IE), Italy (IT), Luxembourg (LU) the Netherlands (NL), Portugal (PT), Sweden (SE). We decide to remove three States that joined the Convention during the period we are looking at: Finland, Cyprus, Turkey

3.2 Variables

Renewal decisions. We use the legal status of PATSTAT to construct the renewal variable. This variable indicates the number of years the patent is renewed. It ranges from 0 which means the patent is granted in a country but not validated, to 16 years which means that the patent is renewed every year up to the statutory limit, i.e., 20 years after filing date. Only a minority of patents are renewed for more than 16 years. This situation happens when the examination period is shorter than 4 years. In these cases, we code them as being renewed for 16 years. We thus assume that the examination period is equal to four years for all the patents in our sample.

We use two sources of information to construct the renewal variable: Information on lapsing and information on renewal. In some countries, the grace period after lapsing for non-payment of renewal fees can be quite long. A lapse event coded in PATSTAT does not necessarily mean a patent expiration. On the other hand, renewal information in PATSTAT is not fully reliable. As noted by Harhoff et al. (2009): ”Following the advice of an EPO expert, information on patent lapses were preferred over renewal information, in case both databases contained conflicting results”. We follow their approach. Ideally, we will evaluate how the results are affected by the way we construct the renewal variable.

Validation decisions. As noted by Hall and Helmers (2019), it is not an easy task to determine from PATSTAT whether a patent has been validated in a country after being granted by the EPO. The legal status of PATSTAT do not provide directly this information because not all the national patent offices record the payment of validation fee. Moreover, not all of them charge validation fee. Here again, we adopt the approach of Harhoff et al. (2009). We assume that the non-validation is indicated as a lapse of the patent in the 365 days following the grant of the patent. As shown in the figure 3, there is a significant variation in validation across countries: from 20% in Greece to almost 80% in Great Britain for EPs in our sample.
**Number of forward citations.** The number of citations is often used to proxy for the value of patent. Many studies find positive correlation between forward citations and the value of patents estimated through other methods (e.g. Trajtenberg, 1990; Lanjouw and Schankerman, 2004). We use here citations 3 years after filing and 5 years after filing to proxy for the scientific contribution of the invention. In the next version of the paper, we will distinguish more precisely citations coming from examiners and applicants, and plan to use the timing of citations in the stochastic patent renewal model.

**Number of claims.** The claims define in technical terms the scope of the protection. The number of claims is usually used as a measure of the patent scope, and we do so as well.

**IPC class.** We follow the existing literature and use the number of IPC (International Patent Classification) subclasses (e.g A101B) for each patent as a measure of the technological breadth of the invention. The IPC subclasses are assigned by the examiner.

**Technology area.** We estimate the model on different samples defined by technology areas. Technology areas are based on the ISI-OST-INPI classification updated by Schmoch (2008) and included in PATSTAT. The patent system is divided into 5 large and 35 more disaggregated technology areas. We look at 5 large technology areas: chemistry, electrical engineering, instruments, mechanical engineering and other fields. We define a patent as belonging to a technology area if at least one IPC code of the patent belongs to this technology area.

### 3.3 Descriptive statistics

Because patent validation and renewal decisions are costly, not all the patent holders choose to validate a granted patent in all the designated countries. Similarly a large proportion of patents is not renewed for the full patent life which is 20 years from patent filing / 16 years...
from grant decision. The table and figures below provide some evidence of differences in the set of validated countries and renewal decisions, as well as differences in total cost of renewal across countries.

The figure 3 shows that there is a substantial number of countries for which the average validation rate is relatively low. In other words, a significant proportion of the patents would be almost worthless in these countries. However, conditional on being validated, Figure 4 indicates that the mean renewal length is not very different across countries. These two figures could suggest that some learning effects are playing a role. The true private value of the patent is discovered through time.

Table 1: Descriptive Statistics for EPs granted between 1998 and 2003, designated in 15 countries

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>St.Dev</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
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<tbody>
<tr>
<td>Renewal and validation:</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Number of validation countries</td>
<td>5.99</td>
<td>4.52</td>
<td>5.00</td>
<td>0</td>
<td>15.00</td>
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<tr>
<td>Number of renewal years in all countries</td>
<td>7.87</td>
<td>4.22</td>
<td>7.00</td>
<td>1.00</td>
<td>16.00</td>
<td>61,968</td>
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<tr>
<td>Patent characteristics</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Forward citations (3 years)</td>
<td>0.3833</td>
<td>1.26</td>
<td>0.00</td>
<td>0.00</td>
<td>34.00</td>
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<tr>
<td>Forward citations (5 years)</td>
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<td>0.00</td>
<td>76.00</td>
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<td>Number of applicants</td>
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<td>1.00</td>
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<td>Number of inventors</td>
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<td>1.00</td>
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<td>Number of claims</td>
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<tr>
<td>Number of IPC classes</td>
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<td>1.51</td>
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<td>1.00</td>
<td>42.00</td>
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</tr>
<tr>
<td>Technology area</td>
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<td></td>
<td></td>
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<tr>
<td>Chemistry (d)</td>
<td>0.37</td>
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<td>0.00</td>
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<tr>
<td>Electrical Engineering (d)</td>
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<td>Instruments (d)</td>
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<td>Mechanical engineering (d)</td>
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<tr>
<td>Other fields (d)</td>
<td>0.07</td>
<td>0.26</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>61,968</td>
</tr>
</tbody>
</table>
Figure 3: Validation decisions for EP granted between 1998 and 2003, designated in 15 countries

Figure 4: Mean renewal year for EPs granted between 1998 and 2003, conditional on being validated
Figure 5: Total renewal costs for a 20-year protection in a given country, in euro 2000
4 Theoretical framework

The validation and renewal decisions at the EPO are complex. We do not attempt to provide an exhaustive model of renewal and validation, but we want to provide insights on these decisions.

4.1 Private value of EP

The main assumption of the model is that firms decide on validation and renewal strategies by comparing the expected returns with the renewal costs, following the deterministic approach of Pakes and Schankerman (1979, 1984) and extended in a stochastic framework by Pakes (1986) and Lanjouw (1998).

Consider an inventor seeking to validate and renew a EP in multiple countries. The granted patent protects a single invention \(i\) indexed by \(i = 1, \ldots, I\) in a country \(j\) indexed by \(j = 1, \ldots, J\).

The return of the invention \(i\) in country \(j\) at period \(t = 1, \ldots, T\) is defined by \(R_{ijt}\) when \(T\) is the statutory duration of the patent. Note that the first period \(t = 1\) corresponds to the first year the patent is granted which usually occurs two to six years after the filing date. The model is deterministic in the sense that the inventor knows perfectly the full sequence of returns from the time the patent is granted. We assume that returns for patents in a country \(j\) depreciate every period at the constant rate \(\delta_j \in (0, 1)\) known by the patent holder and to be estimated. Patent return for an invention \(i\) in country \(j\) at period \(t\) is then:

\[
R_{ijt} = \delta_j^{t-1}R_{ij1} \quad \text{where } R_{ij1} \text{ is the return in the first period}
\]

The renewal cost \(C_{jt}\) in a country \(j\) in period \(t\) with \(t \geq 2\) is known for the full life of the patent. The country-specific validation costs \(C_{j1}\) are not easy to measure (see 2.1), we assume then they are unobserved and parameter to be estimated. These costs are paid one time, once the patent is granted in period \(t = 1\) in order to transfer the EP into a national
The private value of a patent protection is the value to the owner of the patent. This information is not observed by the researcher but is observed by the patent holder. Following Putnam (1997) and Deng (2011) who analyze patent renewal in an international context, the private value of a EP covering an invention \( i \) in a set of countries \( 1, \ldots, J \) is:

\[
V(R; T) = \max_{T_1, \ldots, T_J} \left\{ \sum_{j=1}^{J} \sum_{t=1}^{T_j} \mathbf{1}\{T_j \geq 1\} \beta^{t-1} (\beta \delta_j^{t-1} R_{ij1} - C_{jt}), 0 \right\}
\]

The owner decides for each country \( j \) how many periods \( T_j \in [0, 1, 2, \ldots, T] \) she or he will renew the patent, balancing patent returns with costs. Note that \( T_j = 0 \) means that the patent is not validated in country \( j \) because the patent holder decides not to pay the per-country validation costs \( C_{j1} \). Costs are paid at the beginning of each period whereas returns are received at the end of each period. Returns are discounted by the present discounted factor \( \beta \) which is supposed to be known and fixed to 0.9 following the literature.

### 4.2 Renewal and validation decisions

We can use an assumption of our model and a feature of the renewal data to come up with a way of characterizing the renewal and validation decisions. The assumption of the model is that revenues are (weakly) decreasing over time and the institutional feature is that renewal fees are strictly increasing over time. Therefore, we assume that the validation costs \( C_{j1} \) in period \( t = 1 \) are lower than the renewal fees \( C_{j2} \) in period \( t = 2 \). Together, the assumption and the feature of the data mean that a patent that is worth renewing in period \( t \) is worth more in period \( t - 1 \) than a patent acquired in the same period, but that is not worth renewing in period \( t \). From this, it follows that all those patents that are renewed in period \( t \) must have been at least as valuable in the initial period \( t = 1 \) than the most valuable of those patents that are not renewed in period \( t \). We can then characterize
thresholds for returns in the initial period $t = 1$ which determine the number of years the patent is renewed.

The patent holder renews a patent $i$ in country $j$ in period $t$ as long as:

$$\beta R_{ijt} \geq C_{jt} \implies R_{ijt} \geq \frac{C_{jt}}{\beta^t_{j-1}}$$

We take the log of these expressions and define $r_{ijt} = \log(R_{ijt})$ and $c_{jt} = \log(C_{jt})$. The number of years a patent is renewed in a country $j$ is defined by the discrete observed outcome $y_{ij}$. For each invention $i$ in a country $j$, we observe the categorical variable $y_{ij}$ linked to the unobserved returns $r_{ij}$ by the following rule:

$$y_{ij} = \begin{cases} 
0 & \text{if } -\infty < r_{ij0} \leq c_{j1} - \log \beta \\
1 & \text{if } c_{j1} - \log \beta < r_{ij0} \leq c_{j2} - \log \delta_j - \log \beta \\
\vdots & \\
T & \text{if } c_{jT} - (T - 1) \log \delta_j - \log \beta < r_{ij0} < +\infty 
\end{cases}$$

The log of the initial return in a country $j$ ($r_{ij}$) is the latent variable assumed to be determined by a linear model with a deterministic part observed and a random part unobserved by the researcher:

$$r_{ij1} = X_i' \gamma_j + \epsilon_{ij}$$

where $X_i'$ is a $K$ dimensional vector of observed covariates that include patent characteristics which may affect the quality of the invention. The covariates are the forward citations 3 and 5 years after filing, the number of applicants and inventors, the number of claims and the number of IPC classes. $\gamma_j$ is a vector of parameters to estimate that measure the "source of returns". $\epsilon_{ij}$ is a random component assumed to follow a multivariate normal distribution with mean $\mu \in \mathbb{R}^J$ and covariance matrix $\Sigma \in S_{++}^J$ where $S_{++}^J$ is the
space of symmetric positive definite $J \times J$ matrices. Therefore, unobservable (to the econometrist) parts of return to a patent are correlated across countries. We further assume that model satisfies the exogeneity condition: $E(X_{ij}|\epsilon_{ij}) = 0 \; \forall j$. The assumption that the logarithm of returns is normally distributed is supported by surveys showing that the distribution of patent value is highly right skewed (Gambardella, Harhoff and Verspagen, 2008).

\[
(\epsilon_{i1}, \ldots, \epsilon_{iJ}) \sim N \left( \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_J \end{pmatrix}, \begin{pmatrix} \sigma^2_1 & \rho_{12} & \ldots & \rho_{1J} \\ \rho_{12} & \sigma^2_2 & \ldots & \rho_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1J} & \rho_{2J} & \ldots & \sigma^2_J \end{pmatrix} \right)\]

The correlations in the $\epsilon_{ij}$ allow us to capture the degree of ‘synergies’ between any pair of countries. Nevertheless, estimating the parameters of the covariance matrix for n-countries would imply the estimation of $\frac{n(n+1)}{2}$ parameters which is computationally intensive in our case as the number of countries goes up to 15. (and 38 when we include more recent data).

4.3 Estimation and Identification

Cf Appendix B.

5 Preliminary results

5.1 The effects of UP on private value of patents

The Unitary Patent will have different effects on the private value of patents via territorial scope effects, length effects, cost-efficiency and new inventions. The goal of our work is to estimate more precisely these different effects.
• Increase in the territorial scope of the patent. In the current European Patent System, companies can always reduce the territorial scope of protection by letting their patents to lapse in some countries. In the UP, this decision will not be possible and the lapsing decision will affect the all set of countries. We expect an increase in the territorial scope. This paper intends to estimate the magnitude of this effect by assessing how many EPs patent holders would have apply for a UPs if this alternative was available.

• cost-efficiency: reduce the cost of patenting. A direct effect of the UP is a reduction in the cost of patenting. More specifically, the more countries a classic European Patent would have been validated in, the more cost-effective a Unitary Patent will be. Harmonization of patent systems will help to reduce wasted resources. For large companies which usually validate their EPs in many States, the Unitary Patent may provide cost-efficiency. Alternatively, some patent holders prefer to renew their EPs in only a small subset of States. For them, UP might not appear as a preferable option.

• Affect the length of the patent. Under the new regime, some patents will be renewed longer whereas others will be renewed shorter. Moreover, under the new regimes, some inventions of low value will be now applied for patent protection.

• New inventions: A change in the value of patents leads to changed incentives to invest. As the UP is an option for EP patent holders, it will lead to an increase of those patents that would have been obtained under an EP only regime. To answer the question of how many new inventions (and of what type) would be generated under UP, we need to amend our patent renewal model with a model of patent protection. We are now working on such a model. To illustrate, assume that a patent with private value \( V \) through optimally chosen patent protection has an inventor-specific cost of invention \( c_V \) with a c.d.f. of \( F(c_V) \). Assume further, for simplicity, that the
potential inventors \(i\) (who are capable of coming up with an invention with value \(V\), but cannot invent anything else) have perfect knowledge, i.e., before sinking in the invention cost \(c_{Vi}\), each inventor \(i\) learns \(c_{Vi}\). It would then be the case that the marginal inventor earns zero profits, i.e., \(V = c_{Vi}\) holds. If one was armed with knowledge of \(F(c_V)\), one could answer the question of how many more patents of value \(V\) we would obtain under UP as opposed to a EP-only regime.

More formally, the total private value of an invention \(i\) under the current regime is:

\[
V_i^{EP}(R; \theta) = \sum_{j=1}^{J} \sum_{t=1}^{T_j^*} \beta^{t-1} \left( \beta \delta_j^{t-1} R_{ij1} - C_{jt}^{EP} \right)
\]

where

\[
T_j^* = \max_t \sum_{k=1}^{t} \beta^{k-1} \left( \beta \delta_j^{k-1} R_{ij1} - C_{jk}^{EP} \right)
\]

The total value depends on the estimation of \(\theta\). Similarly, the private value of an invention \(i\) under the Unitary Patent System is:

\[
V_i^{UP}(R; \theta) = \max \left\{ \sum_{t=1}^{T^*} \beta^{t-1} \left( \beta \sum_{j=1}^{J} \delta_j^{t-1} R_{ij1} - C_{jt}^{UP} \right), V_i^{EP} \right\}
\]

where

\[
T^* = \max_t \sum_{k=1}^{t} \beta^{k-1} \left( \beta \sum_{j=1}^{J} \delta_j^{k-1} R_{ij1} - C_{k}^{UP} \right)
\]

Then the private gain/cost of moving from EP to UP is:

\[
\Delta PV = V_i^{UP} - V_i^{EP} = \sum_{j=1}^{J} \sum_{t=1}^{T_j^*} \beta^{t-1} C_{jt}^{EP} - \sum_{t=1}^{T^*} \beta^{t-1} C_{jt}^{UP} + \sum_{j=1}^{J} \sum_{t=T_j^*/T_j^* \neq 0}^{T^*} \beta^{t} R_{ij1} \delta_j^{t-1} \left( \begin{array}{c}
\text{Cost effect} \\
\text{Length effect} \\
\text{Scope effect}
\end{array} \right)
\]

Note that the number of inventions is constant in our case. A next step would be to include
the possibility of new inventions.

5.2 Estimation - preliminary results

For the time being, we study renewal decisions conditional on validation. A next step will be to add the validation decision. As mentioned in the descriptive statistics, validation rates are low for some countries. This becomes a challenge for our model as it is difficult to reconcile large proportion of patents of very low value (not validated) with patents renewed for the full period. The assumption of weakly decreasing return and the fact that validation rates are not high give us unrealistic estimates. One way to rationalize the data might be to introduce a "patent acquisition" cost shock or uncertainty at the validation stage on the patent value. At this stage, we follow the literature and don’t model the validation decisions. Note that we estimate a 'fake' validation cost by removing the first renewal fee. For now, we only present results for the chemical industry.

Preliminary results are summarized in the following figures. The yellow bars display 95% confidence intervals.
Figure 6 shows that the highest initial returns are earned in Germany, The Netherlands, Austria, Great Britain and France. Three of these have the lowest decay rates, meaning that patents in Austria, The Netherlands and Germany lose value faster than in other countries. Figure 7 further shows that four of the five countries (the exception being France) with the highest mean initial returns also have the highest variation in returns, and two of them - The Netherlands and Germany - are in the top-3 of validation costs.
Figure 7: Estimates for $\sigma$ and $C_1$

Figure 8 shows the correlations of country specific unobservable returns with those of Germany and Great Britain. We find high correlations, with the lowest ones being 0.71 in the case of Germany and 0.62 in the case of the UK, both with Luxembourg.
Figure 8: Correlation between returns

5.3 Counterfactual - Preliminary results

Using the parameter estimates, we generate 50,000 patents in the chemical industry and compute the net private value under the current system and under the new system using renewal fees. We decompose the effects as explained in Section 5.1.
Table 2: Counterfactual effects on private value of patents in Euro 2000

<table>
<thead>
<tr>
<th>Statistics</th>
<th>$V^{\text{EP}}$</th>
<th>$V^{\text{UP}}$</th>
<th>$V^{\text{UP}} - V^{\text{EP}}$</th>
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<tr>
<td></td>
<td>Total</td>
<td>Length</td>
<td>Scope</td>
</tr>
<tr>
<td>Q. 10 %</td>
<td>6046</td>
<td>8058</td>
<td>1853</td>
</tr>
<tr>
<td>Q. 25 %</td>
<td>22,367</td>
<td>26934</td>
<td>4386</td>
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<tr>
<td>Median</td>
<td>92,991</td>
<td>100,721</td>
<td>7696</td>
</tr>
<tr>
<td>Q. 75%</td>
<td>379,946</td>
<td>391,430</td>
<td>11,506</td>
</tr>
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<td>Q. 90%</td>
<td>1,424,132</td>
<td>1,438,646</td>
<td>15,064</td>
</tr>
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<td>Q. 95%</td>
<td>3,226,367</td>
<td>3,243,417</td>
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<tr>
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<td>1,074,596</td>
<td>1,082,683</td>
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</tr>
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<td>Min</td>
<td>124</td>
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<td>0</td>
</tr>
<tr>
<td>Max</td>
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<td>2.2B</td>
<td>18102</td>
</tr>
<tr>
<td>N</td>
<td>50,000</td>
<td>50,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

Table 2 reports key findings of our counterfactual exercise. The mean value of a patent increases by 8 000 € from the introduction of UP. More than half of this comes from decreased renewal fees (last column, Cost), and most of the rest from adjusting the optimal length of the patent in those countries where the original EP was taken. Only some 10% of the change in value comes from a change in the geographical scope of the patent.

Further, we see that the distribution of patent value is quite similar under EP and UP. The change in value at the 10th percentile is less than 2 000 €, while at the 90th percentile it is 15 000 €. Notice that currently our calculations "force" patent holders to shift from EP to UP, although UP only presents an option to patent holders.

We have a closer look at the changes in patent value with the help of four graphs. In Figure 9 we show both the total effect (=change in value) and its three constituent parts all as a function of the estimated value of the EP-patent. A first thing to note is that there are very few patents whose value under UP would be smaller than under EP. These seem to be patents whose fees would go up by more than than the increase in gross value. We find that the total effect increases rapidly as a function of the EP-value of the patent. This is driven by the cost effect. The length effect is decreasing in EP-value. The reason for this most likely is that those patents with high EP-value are those that are renewed to
(almost) the maximum and thus there is little scope for those patents to increase their value through a change in length. We find a similar though starker result regarding geographical scope. The likely explanation is similar: the most valuable EP-patents are those with a large geographical coverage.

Figure 9: Decomposition of the effects
5.4 From private value to social value

This section is still very preliminary. Our objective is to assess the effects of introducing the Unitary Patent System in terms of private value for patent holders as well as costs/benefits for the European Patent Office and costs/benefits for consumers.

In order to examine the impact on the consumer surplus, we start with a simple Cournot model with linear demand \( p(Q) = a - bQ \) and constant marginal cost \( c \) in each country. A well-known result is that a monopoly and perfect competition equilibria are characterized by the following profits and consumer surplus:

\[
\Pi_m = \frac{(a - c)^2}{4b} \quad CS_m = \frac{(a - c)^2}{8b} \quad ; \quad \Pi_{pc} = 0 \quad CS_{pc} = \frac{(a - c)^2}{b}
\]

Under the assumptions made, we have a direct mapping between the monopoly profit (invention protected) and the consumer surplus in both monopoly and perfect competition: \( CS_m = \frac{1}{2}\Pi_m \) and \( CS_{pc} = 4\Pi_m \). We can hence estimate welfare both under monopoly -
patent protection - and when it is removed - perfect competition - once we have an estimate of the private value (=monopoly profit) of a patent.

Assuming that the private value computed is a correct proxy for the monopoly profit of a firm, we are able to quantify the effect of the UP on the consumer surplus. In this case, the static profit on an invention $i$ in country $j$ in period $t$ protected by a EP is:

$$\Pi_{ijt}^{EP} = \beta \delta_j^{t-1} R_{ij1}$$

Similarly, the static profit of an invention $i$ in period $t$ protected in all countries by a UP is:

$$\Pi_{it}^{UP} = \beta \sum_{j=1}^{J} \delta_j^{t-1} R_{ij1}$$

The patent-holder benefits from a monopoly power during the full life of the patent. Once the patent lapses, new firms enter the market and the equilibrium is characterized by perfect competition. We assume profits are discounted by $\beta$. The consumer surplus for an invention $i$ under EP and UP regimes is given by the following formulae respectively:

$$CS_i^{EP} = \sum_{j=1}^{J} \sum_{t=1}^{T_j} \frac{1}{2} \beta^{t-1} \Pi_{ijt}^{EP} + \sum_{j=1}^{J} \sum_{t=T_j+1}^{+\infty} 4 \beta^{t-1} \Pi_{ijt}^{EP}$$

$$CS_i^{UP} = \sum_{t=1}^{T^*} \frac{1}{2} \beta^{t-1} \Pi_{it}^{UP} + \sum_{t=T^*+1}^{+\infty} 4 \beta^{t-1} \Pi_{it}^{UP}$$

The total effect of the new regime for a given set of patents is then:

$$\text{Total Effect} = \sum_i V_i^{UP} - V_i^{EP} + \sum_i CS_i^{UP} - CS_i^{EP} + \sum_{t=1}^{T^*} \beta^{t-1} C_t^{UP} - \sum_{j=1}^{J} \sum_{t=1}^{T_j^*} \beta^{t-1} C_{jt}^{EP}$$

$$= \Delta \text{Private Value} + \Delta \text{Consumer Surplus} + \Delta \text{Cost for Patent Office}$$

In ongoing work we are working on how to estimate the welfare increase emanating from an increased incentive to invent.
6 Conclusion

The objective of this paper is to provide a counterfactual analysis of the Unitary Patent system that the EU is about to adopt. We build on the literature that exploits variation in patent renewal fees and decisions to back out the parameters of a structural model of patent value. We extend that literature by allowing for freely correlated returns across countries, and in future work plan to further extend the literature by estimating the validation costs of patents.

Our results show that mean patent returns vary greatly across countries, as do their decay rates and the variation in returns. Yet, the unobservable components of patent value are highly correlated across countries.

We use our estimated parameters to calculate the change in value of existing patents had their owners had the possibility to opt for the Unitary Patent. We decompose the change in value into three component: The length effect captures the change in value that arises from the patentee changing the length of patent protection in those countries where the patentee took out the European Patent; the cost effect is the change in patent renewal fees going from EP to UP; and the scope effect is the increase in value from increasing the geographical scope of the patent. We find that the cost effect is larger on average than either and indeed even the sum of the scope and length effects. We also find that the scope effect increases quickly as a function of the value of the patent in an EP-only regime. The length effect is, unsurprisingly, negatively related to the value of the patent in an EP-only regime: the more valuable patents are those that are renewed for a larger number of years. The scope effect is largest for patents with low EP-only values. Our estimates suggest that almost all patent holders of the EP-only patents in our data would have opted for the UP patent, had that been possible. The estimated benefits from UP would likely be even larger Were we able to incorporate the reduction in translation fees and other transaction costs,

We want to remind the reader that this is ongoing work and our results are very
preliminary. Consequently, there are several possible directions:

- include validation decisions in our empirical part.
- include the patent characteristics in the estimation.
- add litigation costs in the model
- stochastic model
References


Pakes, Ariel, and Mark Schankerman. 1979. “The rate of obsolescence of knowledge, research gestation lags, and the private rate of return to research resources.”


A Fees - amount

A.1 Renewal fees European Patent

All fees are expressed in euros 2000. See table 4.

A.2 Renewal fees Unitary Patent

Table 3: Renewal fees - Unitary Patent

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<thead>
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<th>Year</th>
<th>EUR</th>
<th>Year</th>
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<td>11th year</td>
<td>1460</td>
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<tr>
<td>3rd year</td>
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<td>4th year</td>
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<td></td>
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Note that a 15% reduction in the renewal fees are available for patent holders who file a statement on a licence of right with the EPO
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B Estimation and Identification

The parameter vector to estimate is \( \theta = (C_1, \gamma, \mu, \delta, \sigma, \Omega) \) where \( C_1 \) is a J-dimensional vector of validation costs, \( \gamma \) is a \( J \times K \) dimensional vector of stacked estimates for covariates, \( \mu \) is a J-dimensional vector of mean for initial returns, \( \delta \) is a J-dimensional vector of decay rates, \( \sigma \) is a J dimensional vector of standard deviations for initial returns and \( \Omega \) is a \( \frac{J(J+1)}{2} \) dimensional vector of all error covariances stacked.

Let define the serie of thresholds in (1) using (2) as

\[
\begin{align*}
\kappa_{ij}^0(\theta) &= -\infty \\
\kappa_{ij}^1(\theta) &= c_{j1} - \log \beta - X_i'\gamma_j \\
\kappa_{ij}^2(\theta) &= c_{j2} - \log \delta_j - \log \beta - X_i'\gamma_j \\
&\quad \ldots \\
\kappa_{ij}^k(\theta) &= c_{jk} - (k-1) \log \delta_j - \log \beta - X_i'\gamma_j \\
&\quad \ldots \\
\kappa_{ij}^T(\theta) &= c_{jT} - (T-1) \log \delta_j - \log \beta - X_i'\gamma_j \\
\kappa_{ij}^{T+1}(\theta) &= +\infty
\end{align*}
\]

Note that for a given invention \( i \) in a country \( j \), the unkown thresholds satisfy the condition: \( \kappa_{ij}^0(\theta) < \kappa_{ij}^1(\theta) < \ldots < \kappa_{ij}^T(\theta) \) because the renewal fees are increasing over time. Let \( \phi_{\mu,\Sigma}(\epsilon_{i1}, \ldots, \epsilon_{iJ}) \) be the multivariate probability density function with \( (\epsilon_{i1}, \ldots, \epsilon_{iJ}) \sim \mathcal{N}(\mu, \Sigma) \)

The likelihood function for an invention \( i \) that is renewed \( m_1 \) period(s) in country 1, \( m_2 \) period(s) in country 2, \ldots, \( m_J \) period(s) in country J is the following:
The likelihood involves then the computation of a J-dimensional integral for each invention which is computationally difficult for large $J$. In general, the standard approach is to use numerical simulation techniques based on simulated maximum likelihood (SML) which is sometimes challenging when the outcome dimensions are large. To circumvent this issue, we try two approaches to estimate the model: the composite marginal likelihood (CML) and the univariate ordered probit with computation of generalized residuals.

**Composite marginal Likelihood (CML)** The composite likelihood methods were first introduced by Besag (1975) under the term of pseudo-likelihood and then popularised by Lindsay (1988) as composite likelihood methods. The approach consists of constructing likelihood object based on likelihood of marginal or conditional events. In the recent years, this method has attracted more interest. For instance, Paleti and Bhat (2013) compare SML with the use of a composite marginal likelihood (CML). They show that using SML is cumbersome and prone to simulation errors. Furthermore, they find that CML recovers parameters as well as the SML estimation approach and with a substantially reduced computational cost (see also Bhat, Varin and Ferdous (2010)). This method has been applied widely in the statistics field but has gained little attention in economics and econometrics. Mullahy (2016) propose a composite marginal likelihood approach to estimate multivariate probit models with bivariate probit. In our setting, the approach requires to replace the full likelihood function by a surrogate likelihood constructed from pair-wise bivariate ordered probits. Therefore, the full pair-wise approach of CML requires to evaluate $J \times (J - 1)/2$ pairs.

The standard pairwise CML likelihood function for the invention $i$ is:
\[
L^i_{CML}(\theta) = \prod_{j=1}^{J-1} \prod_{j'=j+1}^{J} \Pr(y_{ij} = m_j, y_{ij'} = m_{j'})
\]

where the probability that an invention \(i\) is renewed \(m_j\) periods in country \(j\) and \(m_k\) periods in country \(k\) is:

\[
\Pr(y_{ij} = m_j, y_{ik} = m_k) = \Pr(k_{ij}^{m_j} \leq \epsilon_{ij} \leq \epsilon_{ij}^{m_j+1} \cap k_{ik}^{m_k} \leq \epsilon_{ik} \leq \epsilon_{ik}^{m_k+1})
\]

\[
= \Phi_2 \left( \frac{k_{ij}^{m_j+1} - \mu_j}{\sigma_j}, \frac{k_{ik}^{m_k+1} - \mu_k}{\sigma_k}, \rho_{jk} \right) - \Phi_2 \left( \frac{k_{ij}^{m_j} - \mu_j}{\sigma_j}, \frac{k_{ik}^{m_k} - \mu_k}{\sigma_k}, \rho_{jk} \right)
\]

\[
- \Phi_2 \left( \frac{k_{ij}^{m_j+1} - \mu_j}{\sigma_j}, \frac{k_{ik}^{m_k} - \mu_k}{\sigma_k}, \rho_{jk} \right) + \Phi_2 \left( \frac{k_{ij}^{m_j} - \mu_j}{\sigma_j}, \frac{k_{ik}^{m_k} - \mu_k}{\sigma_k}, \rho_{jk} \right)
\]

\(\Phi_2\) is the bivariate standard normal cumulative distribution with covariance \(\rho\).

The pairwise marginal likelihood function is then:

\[
L_{CML}(\theta) = \prod_{i=1}^{I} L^i_{CML}(\theta)
\]

Varin and Czado (2009) show that the CML estimator \(\hat{\theta}\) maximizing the likelihood above is consistent. Furthermore, the CML estimator is asymptotically normally distributed. The asymptotic variance-covariance matrix for the estimator is given by the inverse of the Godambe sandwich information matrix (Godambe, 1960) defined by:

\[
V_{CML}(\theta) = [H(\theta)]^{-1}J(\theta)[H(\theta)]^{-1}
\]

with

\[
H(\theta) = E \left[ -\frac{\partial^2 \log L_{CML}(\theta)}{\partial \theta \partial \theta'} \right]
\]

\[
J(\theta) = E \left[ \left( \frac{\partial \log L_{CML}(\theta)}{\partial \theta} \right) \left( \frac{\partial \log L_{CML}(\theta)}{\partial \theta} \right)' \right]
\]

\(H(\theta)\) is then the negative expected Hessian matrix of the composite likelihood and \(J(\theta)\) is the expected product of composite scores. For composite likelihood \(H(\theta) \neq J(\theta)\) which shows a loss of efficiency compared to the MLE.
Univariate ordered probit with generalised residuals. We propose another approach to circumvent the curse of dimensionality problem. We run univariate ordered probit for each country and compute the generalized residuals following the approach of Gourieroux et al. (1987). To the best of our knowledge this method has not been used in the literature and we can offer then a comparison with the CML approach. In later work, this simpler estimation method provides starting values to CML, thereby speeding up estimation greatly.

In the standard univariate model, for an invention $i$, the probability of observing $y_{ij} = m_j$ is defined by:

$$Pr(y_{ij} = m_j) = \Phi\left(\frac{K_{ij}^{m_j+1} - \mu_j}{\sigma_j}\right) - \Phi\left(\frac{K_{ij}^{m_j} - \mu_j}{\sigma_j}\right)$$

where $\Phi$ is the standard normal cumulative distribution function. Parameters $\mu_j, \sigma_j, \delta_j, \gamma_j$ are estimated with $J$ univariate ordered probit.

The log-likelihood maximized for a country $j$ is:

$$\log L_j = \sum_{i=1}^{T} \sum_{t=1}^{T} 1(y_{ij} = t) \log Pr(y_{ij} = t)$$

The multivariate ordered probit can be break into $J$-univariate ordered probit where we compute the covariance matrix through residuals based on the methodological work of Gourieroux et al. (1987). Using the estimates $\hat{\mu}_j, \hat{\sigma}_j, \hat{\delta}_j, \hat{\gamma}_j$, the residuals for a patent $i$ in country $j$ are computed as (result given by Vella (1993) for the ordered probit and derived in appendix C):

$$e_{ij} = \begin{cases} 
- \phi\left(\frac{\hat{c}_{ij,1} - \log \beta_i - X_i'\hat{\gamma}_j - \hat{\mu}_j}{\hat{\sigma}_j}\right) & \text{if } y_{ij} = 0 \\
\Phi\left(\frac{\hat{c}_{ij,1} - \log \beta_i - X_i'\hat{\gamma}_j - \hat{\mu}_j}{\hat{\sigma}_j}\right) - \phi\left(\frac{\hat{c}_{ij,k+1} - k \log \hat{\delta}_j - X_i'\hat{\gamma}_j - \hat{\mu}_j}{\hat{\sigma}_j}\right) & \text{if } y_{ij} = k; \ 1 \leq k \leq T \\
\phi\left(\frac{\hat{c}_{ij,k+1} - k \log \hat{\delta}_j - X_i'\hat{\gamma}_j - \hat{\mu}_j}{\hat{\sigma}_j}\right) - \Phi\left(\frac{\hat{c}_{ij,\tau} - (\tau-1) \log \hat{\delta}_j - X_i'\hat{\gamma}_j - \hat{\mu}_j}{\hat{\sigma}_j}\right) & \text{if } y_{ij} = \tau \\
1 - \Phi\left(\frac{\hat{c}_{ij,\tau} - (\tau-1) \log \hat{\delta}_j - X_i'\hat{\gamma}_j - \hat{\mu}_j}{\hat{\sigma}_j}\right) & \text{if } y_{ij} = T 
\end{cases}$$
where $\phi$ is the standard normal pdf and $\Phi$ the standard normal cdf.

**Identification.** In the standard ordered probit, unrestricted variance parameters are unidentified (Greene and Hensher, 2010). In our model renewal fees provide information on scaling of the latent variable. In other words, if the latent variable $r_{ij}$ as well as the unknown parameters to estimate are scaled by the same positive value, the observed outcome $y_{ij}$ will not be preserved because of the presence of renewal fees that are fixed. Therefore, unlike in the standard ordered probit, no restriction on variance parameter is needed. Furthermore, we assume $X_i$ does not contain a constant term so the standard normalization $\mu = 0$ becomes unnecessary in this case.
C Generalised residuals for ordered probit

Gourieroux et al. (1987) show that the generalized residuals for non-linear models can be obtained by deriving the score with respect to the intercept evaluated at the maximum likelihood estimates. In this part, we derive the generalized residuals for the ordered probit model.

We follow the presentation of the ordered probit by Cameron and Trivedi (2005). The single latent variable is:

\[ y_i^* = X_i' \beta + u_i \]

where \( X \) include an intercept. Note that in our model, the role of the intercept is played by the mean of the initial return. The general form of an m-alternative ordered model is:

\[ y_i = j \quad \text{if} \quad \alpha_{j-1} < y_i^* \leq \alpha_j \]

where \( \alpha_0 = -\infty \) and \( \alpha_m = +\infty \).

Then,

\[
Pr(y_i = j) = Pr[\alpha_{j-1} < y_i^* \leq \alpha_j] \\
= Pr[\alpha_{j-1} - X_i' \beta < u_i \leq \alpha_j - X_i' \beta] \\
= F(\alpha_j - X_i' \beta) - F(\alpha_{j-1} - X_i' \beta)
\]

where \( F \) is the cumulative distribution function of \( u_i \). The estimates of the regression parameters \( \hat{\beta} \) (including the estimate for the constant \( \hat{\beta}_0 \)) and the estimate threshold parameters \( \hat{\alpha}_1, \ldots, \hat{\alpha}_{m-1} \) are obtained by maximising the following log-likelihood:

\[
\mathcal{L} = \log L_N = \sum_{i=1}^{N} \sum_{j=1}^{m} y_{ij} \log \left( F(\alpha_j - X_i' \beta) - F(\alpha_{j-1} - X_i' \beta) \right)
\]
The score with respect to the intercept is:

$$\frac{\partial L}{\partial \beta_0} = \sum_{i=1}^{N} \sum_{j=1}^{m} y_{ij} \frac{F'(\alpha_{j-1} - X_i' \beta) - F'(\alpha_j - X_i' \beta)}{F(\alpha_j - X_i' \beta) - F'(\alpha_{j-1} - X_i' \beta)}$$

In the case of the ordered probit model, $F$ is the normal standard cumulative distribution function $(\Phi)$ and $F'$ is the normal standard probability density function $(\phi)$. The generalised residuals for an observation $i$ and the outcome $j$ is then:

$$r_{ij} = \frac{\hat{\phi}(\hat{\alpha}_{j-1} - X_i' \hat{\beta}) - \hat{\phi}(\hat{\alpha}_j - X_i' \hat{\beta})}{\hat{\Phi}(\hat{\alpha}_j - X_i' \hat{\beta}) - \hat{\Phi}(\hat{\alpha}_{j-1} - X_i' \hat{\beta})}$$
D Monte Carlo simulations

To analyze the sample properties of CML estimator (CMLE) and univariate with generalised residuals estimator (UOPGRE), we implement Monte-Carlo simulations. The results of our simulation are provided in table 5. They indicate that the UOPGRE performs quite well in recovering the true parameters and is faster than the CMLE. We simulate 10,000 patents covering up to 5 countries with the true parameters below. The observable part of the return is assumed to be one variable drawn from a normal distribution $\mathcal{N}(2, 3)$. Using these parameters, give us different range of histograms for renewal in figure 11.

The performance of the method is assessed by per-parameters mean squared errors (MSE). MSE are the squared differences between the true value of the parameters and the estimation, averaged after 500 replications (to do). The standard errors for the covariances of UOPGRE are estimated by bootstrapping (to do).
Figure 11: Renewal and validation for simulated EP
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<th>Std.Err</th>
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Time (sec) | 2450.9 | 5.5. | . | . | .