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A Characteristic Approach to Technology and Technological Change

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Abstract

We develop a characteristic-based model for the endogenous determination of technical coefficients in a linear economy. It is suitable to be interpreted in terms of a knowledge based view of technolog

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1. Introduction

In recent years a great deal of attention has been paid to the construction of models describing the dynamics of an economy endogenously driven by a knowledge originated change in technology. Interesting as it may be, this literature is unsatisfactory as it considers the relationship between knowledge and technology as exogenously given. As a matter of fact, the construction of a knowledge-based theory of technology is considered still an important open issue (Dosi and Nelson (2010), Dosi and Grazzi (2010)). Recent works by von Tunzelmann (2003) and Dosi and Grazzi (2006) provide interesting suggestions as well as conceptual contributions to this issue: [1] von Tunzelmann (2003)[2] extends Sen's capability approach (see Sen (1985)) to production theory to provide a conceptual foundation to dynamic capability theory (see, for Teece, Pisano and Shuen (1995)). Since the capability approach is a characteristic-based approach (Lancaster (1966a))[3] which emphasizes the role of knowledge and skills in

[1] An additional paper worth of mentioning is Auerswald, Kauffman, Lobo and Shell (2000) which develops a model of technology along the recipe approach. However, the model is essentially macroeconomic and does not deal with the interrelation between techniques, prices and distribution.

[2] See also von Tunzelmann and Wang (2007).

[3] Early works on characteristic-based approaches to production are represented by the literature on engineering approach to production function (see, e.g. Chenery (1949), (1953), Marsden, Pingry and

extracting characteristics, von Tunzelmann's work represents a very promising for the construction of a model describing the dynamics of an economy originated by growth of knowledge. [4] Dosi and Grazzi (2006) point out the existence of a theoretical gap between the procedure-centered representation of technology (see, e.g., Winter (1968), Nelson and Winter (1977))) and the input/output-centered representation of technology, and they use the former approach to justify, within a Sraffa-Leontief approach, the "stylized facts" of asymmetries in productivities across and within firms, and heterogeneity of relative input intensities and their persistence over time (Dosi and Grazzi (2006, p. 180)). Both von Tunzelmann and Dosi and Grazzi explicitly refer to "classical" linear production model *à la* von Neumann-Leontief-Sraffa as the most appropriate analytical framework within which to develop a model representing the overall economy with a microfounded analysis of production along the proposed view (von Tunzelmann and Wang (2007, p. 208), Dosi and Grazzi (2006, p. 196)). Although conceptually very interesting and suggestive, these works do not provide any formal analysis of the endogenous determination of technical coefficients and associate prices

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Whinston (1974)) and more recent works on technical change like Triplett (1985). This literature, however, has never developed a systematic analysis of this approach and, in particular, has never emphasized the role of knowledge.

[4] Lancaster (1966b) considers explicitly the role of knowledge in determining the technology of extraction of characteristics from final goods.

and distribution; so we are still far from a rigorous knowledge-based theory of *explaining* technical coefficients, their distribution and dynamics, and associated prices and distribution (see also Dosi and Nelson (2010), Dosi and Grazzi (2010)).

In this paper we extend Lancaster-Sen's characteristic approach to a linear production model *à la* von Neumann-Leontief-Sraffa and we show that this model is able to endogenously determine technical coefficients, prices and distribution; in addition, we show that the model is able to deal, although in a very stylized way, with the heterogeneity of technical coefficients and with the dynamics of the whole economy as driven by the evolution of knowledge. By doing so, the paper provides a first contribution to fill in the gap in von Tunzelmann, Dosi and Grazzi's informal analysis by showing that the formalisation of von Tunzelmann's intuition of extending the capability approach to production theory provides an initial step towards a rigorous "procedural" foundation of the input-output representation of technology and of the evolution of the entire economy itself as endorsed by von Tunzelmann (2003) and Dosi and Grazzi (2006) themselves. For the reasons just said, our model can be interpreted also as providing a theoretical foundation to the empirical analysis of the "stylized facts" in Dosi and Grazzi (2006).

By following von Tunzelmann and Dosi and Grazzi's suggestion, the "aggregate representation of technological interdependencies" is carried out within the classical approach as formalized by Sraffa (1960) (see, e.g. Kurz and Salvadori (1998), Bidard (2004)) where only the price side of the economy and its evolution over time are considered.[5] A more complete analysis should include also the demand side of the economy. This extension could naturally be carried out by using the characteristic approach as usually done in consumer theory. We confine our attention to the price side of the economy for the sake of simplicity and also for the still unsatisfactory state of demand theory in linear production models (for developments taking into account demand, see D'Agata (2010)).

The paper is organized as follows. Next section contains a description of the view of technology we propose. Section 3 develops the model within a partial equilibrium framework. Section 4 provides a general equilibrium linear multi-sectoral model where prices, distribution and technical coefficients are simultaneously determined. Section 5 introduces a very simple model of adaptive dynamics in the previous model in order to deal with dynamics of technical coefficients driven by the growth of firms' capabilities. Section 6 provides some final comments.

[5] Besides to the articles by Dosi and Grazzi and von Tunzelmann and Wang, the use of the classical approach for analyzing issues concerning heterogeneous firms, technical change and structural dynamics has been forcefully endorsed, among others, by Pasinetti (1981), (1993) and Landesmann (1988), and it is still an area of active research (see, e.g. Quadrio-Curzio (1986), Bidard (2010)).

2. Technology

In this section we develop intuitively a characteristic approach to technology which is particularly apt at being integrated within the "procedural" approach proposed by the evolutionary theory. According to this approach a production technique is conceived as being determined by the firm's ability "to do something" (see, e.g. Winter (1968)), or by firm's"deep craft" (Arthur (2009)): specifically, in our case, and paralleling Sen's capability approach, a production technique is conceived as being determined by firm's ability to extract (technical) characteristics from inputs.

Assume that there are n produced goods used as inputs, only one non produced input (labour) indicated by n+1, m technical characteristics and F_i firms in industry i, $i \in N = \{1, 2, ..., n\}$. Indicate by N_i the index set of firms in industry i. Figure 1 illustrates intuitively the productive process to produce *one* unit of good i, $i \in N$, by firm i_f, $i_f \in N_i$. The choice variables of firms are the quantities used of the n+1 inputs; however, production of the output is assumed to be generated by the amount of *technical* characteristics extracted from the inputs. Based upon the "procedural approach" to production, the extraction of characteristics is interpreted to be determined by the "rules" that firms follow in using inputs, which are in turn determined by firms' (static and dynamic) "capabilities" (see, e.g. Richardson (1972), Zander and Kogut (1993), Teece, Pisano and Shuen (1995)).

Assumption 2.1. In order to produce y_i units of good i, $y_i \in \mathbb{R}_+$, $i \in N$, it is necessary to use at least $y_i c^i_k$ units of characteristic k (k = 1,2,…, m), with $c^i_k \geq 0$. Moreover, for every good index i, there is at least one characteristic index k, such that $c^i_k > 0$.

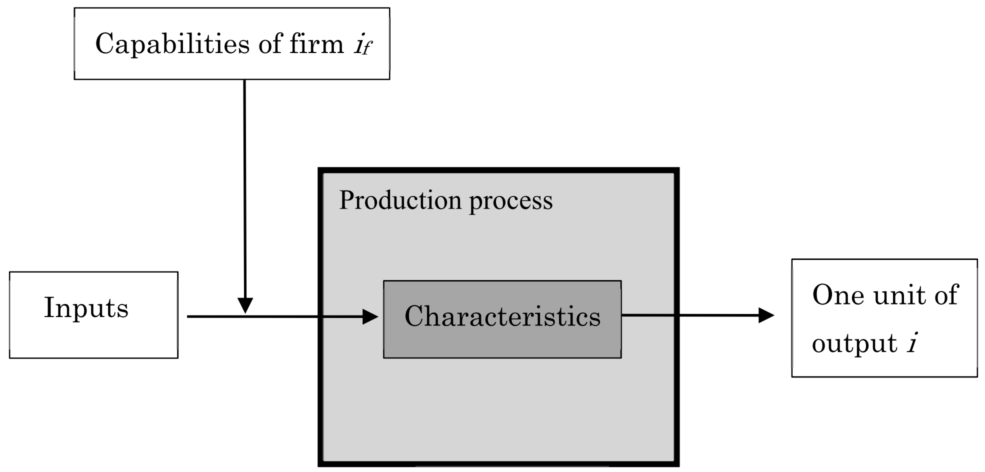

Figure 1

The m-dimensional non-negative (column) vector of characteristics $c^i = \left(c^i_k\right)$ required to produce *one* unit of good i is called the vector of *necessary characteristics* to produce good i. Assumption 2.1. means that the technology to produce output from technical characteristics exhibits constant returns to scale; moreover, it requires that the vectors of necessary characteristics c^i are all semipositive. A possible interpretation of the necessary characteristic vectors is that they represent constraints set by "nature". In principle, more complex views of nature in setting these constraints can be conceived (see Section 6).

As said previously, the characteristics used to produce each good are extracted from labour and the n produced goods, according to firms' capabilities. We indicate by $b_{kj}^{i_f}(\boldsymbol{\sigma})$ the amount of characteristic k that firm $i_f \in N_i$ operating in sector i is able to extract from *one* unit of input *j*, $j \in N_+ = \{1,2,...,n,n+1\}$, for a given capability profile in the economy $\sigma = (\sigma^{i_1},...,\sigma^{i_{F_i}})_{i\in N} \in K = \prod_{i\in N}(K^{i_1} \times ... \times K^{i_{F_i}})$ where K^{i_f} is the knowledge space of firm i_f.[6] Without loss of generality, we assume that all sets K^{i_f} are subsets of a k-dimensional Euclidean space.[7] The m-dimensional non-negative (column) vector $b_j^{i_f}(\boldsymbol{\sigma})$ of characteristics extracted by firm i_f from one unit of input j is called the vector of *extracted characteristics* from input j by firm i_f. Matrix $B^{i_f}(\boldsymbol{\sigma}) = \left(b_j^{i_f}(\boldsymbol{\sigma})\right)_{j\in N_+}$ is called the *extraction matrix* of firm i_f and describes the *technology of firm* i_f associated to the capability profile σ.[8] Given a configuration of firms' capabilities in all industries $\sigma \in K$, a *technology* is a collection of $\mathfrak{N} := \sum_{i=1}^{n} F_i$ matrices $T(\boldsymbol{\sigma}) = \left(B^{i_1}(\boldsymbol{\sigma}),...,B^{i_{F_i}}(\boldsymbol{\sigma})\right)_{i\in N} \in \mathbb{R}_+^{(n+1)\times\mathfrak{N}}$, one for each sector, associated with a

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6 It is worth noticing that we allow for external inter-firms and inter-industry spillovers in line with empirical and theoretical literature (Lundvall (1992), Nelson (1993)).

7 Olsson (2000) develops a model of knowledge where the knowledge space is formalised as a subset in a finite dimensional metric space.

8 It is reasonable to assume that "nature" sets also a vector $\tilde{b}_j^i$ of *maximum* amount of characteristics that can potentially be extracted by one unit of each input, hence $b_j^{i_f} \in \left\{x \in \mathbb{R}_+^m \middle| x \leq \tilde{b}_j^i\right\}$. This implies that in each industry the set of feasible extraction matrices is bounded.

profile σ of firms' capabilities in the whole economy.

Assumption 2.2. For every $\sigma \in K$, $b_j^{i_f}(\boldsymbol{\sigma}_{i_f}) \in \mathbb{R}_+^m$ for every $i_f \in N_i$, $i \in N$, and for every $j \in N_+$. Moreover, from x_{ji_f} units of input j used by firm i_f, $x_{ji_f} \geq 0$, firm i_f is able to extract vector $x_{ji_f} b_j^{i_f}(\boldsymbol{\sigma})$ of characteristics.

Assumption 2.2. maintains that, given their capabilities, firms have constant returns to scale in extracting characteristics from inputs. Also for the extraction technology it is possible to conceive more general approaches (see Section 6).

We conclude this section by presenting an example which translates the "art of pin-making" as described by Babbage (1832, p. 133 ff.) into our language. The aim of this example is to show in a concrete case that our model of technology can easily incorporate a more detailed description of the relation between knowledge, procedures and technology.

Example. Pins are produced by using brass and labour as inputs. The latter exerts seven kinds of "mechanical work" or "tasks" on brass ("wire-drawing", etc.). So the space of technical characteristics is an eight-dimensional Euclidean space: the first dimension measures the amount of "material" (brass) used in the process, the remaining seven dimensions measure the amounts of the seven kinds of tasks (measured in "mechanical works") involved in production. Suppose that c = (1, 1, 1, 1, 4, 2, 2, 1), then in order to produce 1 unit (for example, 1 kg.) of pins it is necessary to provide at least 1 unit (1 kg.) of "material" (brass), 1 unit (resp. 2, resp. 4 units), for example joule, of task of type 1, 2,3 and 7 (resp. 5 and 6, resp. 4). The pin-making technology is defined by a 7×2 extraction matrix $B = (b_1,b_2)$, whose first (resp. second) column b_1 (resp. b_2) indicates the extraction technology for one unit of brass (resp. labour). In order to determine the extraction technology, we need a more precise description of the "recipe" used to produce pins. To this aim, we adopt the following assumptions, which are consistent with Babbage's view: (i) the amount of wasted material is proportional to the average number of tasks (*AT*) carried out by each worker, for example, 0.1*AT*; (ii) the amount of hourly work exerted by each worker is inversely related to the number of tasks (*T*), for example $10 - T$; finally, (iii) each worker employed in more than one task distributes uniformly his/her total work on each task. The last assumption means that if a worker carries out *T* tasks, then the hourly work provided in each task is $(1 - T)/T$. Thus, if, for example, we employ one worker per one hour in all seven tasks, by (i), $b_1 = (0.3,0,0,\ldots,0)^T$ and, by (ii) and (iii), $b_2 = (0, 3/7, 3/7, \ldots, 3/7)^T$. By contrast if we employ seven workers for 1/7 of hour each and in only one task each, then $b_1 = (0.9,0,0,\ldots,0)^T$ and $b_2 = (0, 9/7, 9/7, \ldots, 9/7)^T$.

3. Endogenous determination of technical coefficients

In this section, we determine how, given a capability profile $\sigma \in K$ and associate technology of firm i_f, $B^{i_f}(\boldsymbol{\sigma}) = \left(b_j^{i_f}(\boldsymbol{\sigma})\right)_{j \in N_+}$, this firm determines its input coefficients on the basis of a current price vector and distributive variables (i.e. wage and profit rates).

Given a capability profile $\sigma \in K$, set $S^{i_f}(\sigma) = \{x \in \mathbb{R}_+^{n+1} \mid B^{i_f}(\sigma) \cdot x \geq c^i\}$ is the set of *feasible methods of production* of firm i_f. An $(n+1)\times\mathfrak{N}$-tuple of feasible methods of production, one for each firm, $\boldsymbol{\tau}(\boldsymbol{\sigma}) = (x^{i_1}, ..., x^{i_{F_i}})_{i \in N} \in S(\boldsymbol{\sigma}) = \prod_{i \in N}(S^{i_1}(\boldsymbol{\sigma}) \times .... \times S^{i_{F_i}}(\boldsymbol{\sigma}))$ is said a *technique* associated to capability profile σ.

Assumption 3.1. For every $\sigma \in K$ and for every $i_f \in N_i$ and $i \in N$: $S^{i_f}(\sigma) \neq \emptyset$. Moreover, set $S^{i_f}(\sigma)$ is known to firm i_f.

The first part of Assumption 3.1 is equivalent to saying that if c_j^i is positive, than the j-th row of matrix $B^{i_f}(\sigma)$ is semi-positive. The last part of Assumption 3.1. is obviously particularly strong and in the Appendix we relax it by allowing firm i_f to adaptively discover part or the whole set $S^{i_f}(\sigma)$ of feasible methods of production. Although this analysis fits particularly with the overall logic of this paper, it is developed separately in order to keep the analysis of determination of prices and distribution, and its evolution as simple as possible.

We associate at each technique $\boldsymbol{\tau}(\boldsymbol{\sigma}) = (x^{i_1}, ..., x^{i_{F_i}})_{i \in N} \in S(\boldsymbol{\sigma})$ its "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(\boldsymbol{\sigma})) = (x^1, ..., x^n)_{i \in N} \in \mathbb{R}_+^{(n+1)\times n}$ defined as follows: $x^i = \sum_{i_f \in N_i} \beta_{i_f} x^{i_f}$ with $\beta_{i_f} > 0$ and $\sum_{i_f \in F_i} \beta_{i_f} = 1$. In words, the "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(\boldsymbol{\sigma}))$ is obtained from the initial technique $\boldsymbol{\tau}(\boldsymbol{\sigma})$ by an "aggregation rule" consisting in taking the average of methods of production in each sector with weights $\boldsymbol{\beta}_{i_f}$s. Notice also that an "average" technique is represented by a usual input-output matrix $A = \begin{bmatrix} x_{11} & ... & x_{1n} \\ ... & ... & ... \\ x_{n1} & ... & x_{nn} \end{bmatrix}$ and by a vector of labour coefficients $l = \left[x_{n+11} ... x_{n+1n}\right]$.

Let the price vector $p = (p_1, \ldots, p_n)$, the profit rate r and the wage rate w be given. Vector $p_r = ((1 + r)p_1, \ldots, (1 + r)p_n, w)$ will be called the *extended* price vector. Given the extended price vector $p_r \in \mathbb{R}_+^{(n+1)}$, a capability profile $\sigma \in K$ and the associated technology $T(\sigma)$, firm i_f is assumed to choose its input coefficients in such a way to minimize its unit production cost, i.e. it is assumed to choose its method of production x^{i_f} in such a way to solve the following minimization problem.[9]

Problem 3.1 (i_f): $\min p_r \cdot x^{i_f}$ s.t. $B^{i_f}(\sigma) \cdot x^{i_f} \geq c^i$ with $x^{i_f} \in \mathbb{R}_+^{n+1}$.

Lemma 3.1. *For every* $i_f \in N_i$ *and* $i \in N$, *Problem* 3.1(i_f) *has a solution.*

Proof. The feasible set of Problem 3.1.(i) is set $S^{i_f}(\sigma)$ which is nonempty by

[9] According to the definition of extended price vector, it turns out that wages are paid *ex post*.

Assumption 3.1. and clearly closed. Let β be the minimum non-zero element of all matrices $B^{i_f}(\sigma)$ and γ be the maximum element of all vectors c^i. The ratio γ/β is positive from Assumptions 2.1 and 2.2.. It is possible to show that any solution to Problem 3.1.(i_f) restricted to the compact set $\bar{S}^{i_f}(\sigma) = S^{i_f}(\sigma) \cap \left\{ x \in \mathbb{R}^{n+1}_+ \middle| 0 \le x_j \le 2\gamma/\beta, j \in N_+ \right\}$ is a solution to Problem 3.1.(i_f). Thus, the assertion follows from the continuity of the objective function.♦

Any solution $\begin{pmatrix} a^{i_f} \\ l_{if} \end{pmatrix}$ to Problem 3.1.(i_f) is obviously a vector of technical coefficients of firm i_f, given an extended price vector p_r and given the current capability profile σ of the economy. Coefficient a_{ji_f} is the usual *input coefficient* of good j for producing good i of firm i_f, while l_{i_f} is its *labour coefficient*. A $(n+1)\times\mathfrak{N}$-tuple of solutions, one for each firm, $\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1},\ldots,a^{i_{F_i}} \\ l_{i_1},\ldots\ldots,l_{i_{F_i}} \end{pmatrix}_{i\in N}$ is called an *optimal technique* associated to the capability profile σ, to price p and distributive variables w and r. From technique $\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma})$ we can determine the "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma})) = \begin{pmatrix} a^1,\ldots,a^n \\ l_1,\ldots\ldots,l_n \end{pmatrix}$ by using the "aggregation rule" previously mentioned; i.e. $a^i = \sum_{i_f \in N_i} \beta_{i_f} a^{i_f}$ with $\beta_{i_f} > 0$ and $\sum_{i_f \in F_i} \beta_{i_f} = 1$.

Example. It is immediate to check that, in the Example in Section 2, whatever the price vector, in case of employment of one worker the technical coefficient of brass is always equal to 10/17, while the technical coefficient of labour is equal to 28/3, while in case of seven labourers specialised in each process, the brass coefficient is equal to 10/11 and the labour coefficient 28/9.

4. Techniques, prices and distribution in a linear production model

Given a capability profile $\sigma \in K$ and associate technologies $T(\boldsymbol{\sigma})$, and given an extended price vector $p_r = \left(\left(1 + r\right) p_1, \ \ldots, \ \left(1 + r\right) p_n, \ w\right) \in \mathbb{R}_+^{n+1}$, from Problems 3.1.($i_f$), $i_f \in N_i$ and $i \in N$, an optimal technique $\boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma})$ is determined, which is assumed to be ruling in the economy. This technique, once adopted, will determine prices and distribution. If the price vector and distribution associated to this optimal technique are the same as the initial ones, then the price vector and the distribution variables are consistent with the optimal technology, and in this sense the economy can be considered in "equilibrium". In this section we shall deal with the issue of existence of this "equilibrium". In dealing with this issue, the profit rate will be considered as exogenously given at level $r \geq 0$. By standard results, our analysis holds true also in case in which the wage rate is the exogenous variable.

Given the heterogeneity of firms in each sector, and following a standard procedure in linear models of production, we determine the price vector and the wage rate on the basis of an "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma}))$ of the optimal technique $\boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma})$.

To the "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma}))$, we associate the non-negative price vector $p' = (p_1',...,p_n')$ and the non-negative wage rate w' defined by the equation: $(1+r)p'A+w'l = p'$. Prices p' are called (*production* or *long period*) *prices associated with the* (*"average"*) *technique* $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma}))$, while the wage rate w' and the (given) profit rate r are called, respectively, the (*long period*) *wage and profit rates associated with the* (*"average"*) *technique* $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma}))$. At the price vector p' and wage rate w', firm i_f will obtain positive, zero or negative extraprofits, according to whether its unit production costs are lower, equal or higher than the unit cost of the "average" method of production in that industry. These extraprofits ρ_{i_f}, which are substantially Sraffa's quasi-rent (Sraffa (1960, Chapter XI)), are defined by the equation $(1+r)p'\cdot a^{i_f} + w'l_{i_f} + \rho_{i_f} = p_i'$.

Once the price vector p' and wage rate w' are considered, it is natural to ask whether they are equal to the initial values, i.e. whether $p = p'$ and $w = w'$. If the answer is yes, then technique $\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma})$ is, obviously, optimal also at prices p', w' and r (or, equivalently, p w and r). Hence, at the capability profile σ, technique $\tau(p,w,r;\sigma)$ and values p', w' and r are self-consistent; if the answer is no, then once the extended price vector p_r' rules in the economy, technique $\boldsymbol{\tau}(p,w,r;\boldsymbol{\sigma})$ may be displaced by some other optimal technique, and an iterative process of adjustment of prices and wage rate, and techniques may be activated. So, as already said at the beginning of this section, the former case illustrates a case of "equilibrium" in terms of prices and chosen techniques. More formally, given a rate of profit $r \geq 0$ and a capability profile $\sigma \in K$, technique $\boldsymbol{\tau}(\boldsymbol{\sigma})$ is said an *r-efficient technique* if its associated non-negative wage rate w and non-negative price vector p are such that $\boldsymbol{\tau} = \boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma})$, i.e. for every $i \in N$ and for every $i_f \in N_i$ vector $\begin{pmatrix} a^{i_f} \\ l_{i_f} \end{pmatrix}$ is a solution to Problem 3.1.(i_f) with respect to the associated extended price vector p_r= ((1+r)p_1, …, (1+ r)p_n, w).[10] From what has been said previously, the following is evident:

Fact 1. Technique $\boldsymbol{\tau}(\boldsymbol{\sigma})$ is an r-efficient technique at $r \geq 0$ if $\boldsymbol{\tau}(\boldsymbol{\sigma}) = \boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma})$ where p, and w are the price vector and wage rate (at profit rate r) associated to the "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(\boldsymbol{\sigma}))$; i.e. $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(\boldsymbol{\sigma}))$ satisfies the condition: $(1+r)\, pA + wl = p$.

In this section we shall show that, given a capability profile $\sigma \in K$ with associate technology $T(\sigma)$, for every non-negative profit rate r in some interval the iterative process of adjustment generated by solving Problem 3.1.(i_f) in case techniques are not r-efficient converges towards an r-efficient technique.

Assumption 4.1. Vector $d = (1, ..., 1)^T \in \mathbb{R}^{n+1}$ is the *numéraire*, that is $\sum_{i=1}^{n} p_i + w = 1$.

[10] It is easy to show that the concept of r-efficient technique is equivalent to the concept of long-period (or dominant) technique (at the rate of profit r) usually used by the literature on the classical approach (Kurz and Salvadori (1998), Bidard (2004)).

Vector d will be used either with the economic meaning of the bundle used as standard of value and also simply as the vector whose elements are all equal to one. The context will make it clear the appropriate interpretation for vector d.

Assumption 4.2. For every σ∈K and every technique $\boldsymbol{\tau}(\boldsymbol{\sigma}) = (x^{i_1}, ..., x^{i_{F_i}})_{i \in N} \in S(\boldsymbol{\sigma})$, one has $x^{i_f}_{n+1} \in \mathbb{R}^n_{++}$.

Assumption 4.2. means that the labour input coefficients are always strictly positive. This assumption is ensured, for example, if for all industries i and every firm i_f there is a characteristic k so that $c^i_k > 0$ and $b^{i_f}_{kj} = 0$ for all inputs j except for $j = n+1$.

The following result is an immediate consequence of Assumption 4.2.:

Fact 2. For every σ∈K and every extended price vector $p_r \in \mathbb{R}^{n+1}_+$, the "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma}))$ associated to an optimal technique $\boldsymbol{\tau}(p, w, r; \boldsymbol{\sigma})$ satisfies the condition: $l \in \mathbb{R}^n_{++}$.

Lemma 4.2. *Let* $\boldsymbol{\tau}(\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}, ..., a^{i_{F_i}} \\ l_{i_1}, ..., l_{i_{F_i}} \end{pmatrix}_{i \in N}$ *be any technique. Suppose that p* = (p_1, ...,p_n) *and w* > 0 *are associated with the "average" technique* $\tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix}$ *and suppose also that at the extended price* p_r = ((1+ r)p_1, ..., (1+r)p_n, w) *technique τ is not optimal while technique* $\hat{\boldsymbol{\tau}}(\boldsymbol{\sigma}) = \begin{pmatrix} \hat{a}^{i_1}, ..., \hat{a}^{i_{F_i}} \\ \hat{l}_{i_1}, ..., \hat{l}_{i_{F_i}} \end{pmatrix}_{i \in N}$ *is optimal. If p'*=(p_1', ...,p_n') and *w' are the price vector and wage rate associated with the "average" technique*

$\tilde{\tau}(\hat{\tau}(\sigma)) = \begin{pmatrix} \hat{A} \\ \hat{l} \end{pmatrix} = \begin{pmatrix} \hat{a}^1,...,\hat{a}^n \\ \hat{l}_1,.....,\hat{l}_n \end{pmatrix}$, *then* $w' > w$.

Proof. By assumption, $(1+r)p\hat{a}^{i_f} + w\hat{l}_{i_f} \leq (1+r)pa^{i_f} + wl_{i_f}$ for every $i_f \in N_i$ and every $i \in N$, moreover, at least one of the previous inequalities must be satisfied as a strict inequality. Thus, the "average" methods of production satisfy the inequalities $(1+r)p\hat{a}^i + w\hat{l}_i \leq (1+r)pa^i + wl_i$, $i \in N$, with at least one inequality satisfied as strict inequality. The assertion then follows from standard results (see Kurz and Salvadori (1998), Bidard (2004)). ♦

By taking into account Assumption 4.2., the following general result is standard in the literature on linear economies (see Bidard (2004)):

Fact 3. For every technology $\tau(\sigma)$, every associated average technology $\tilde{\tau}(\tau(\sigma))$ and every profit rate $r \in \left(-1, \frac{1-\lambda(A)}{\lambda(A)}\right)$, a positive wage rate w and a positive price vector p such that $p = (1+r)pA+wl$ are uniquely determined, where $\lambda(A)$ is the Frobenius root of matrix A. Moreover, $w = \frac{1}{1+l[I-(1+r)A]^{-1}d}$ and $p = wl[I-(1+r)A]^{-1}$.

Notice that in the previous result the interval $\left(-1, \frac{1-\lambda(A)}{\lambda(A)}\right)$ may be a subset of the negative real numbers, so the rate of profit may be negative. In order to avoid negative profit rates we introduce the following assumption:

Assumption 4.3. For every $\sigma \in K$ there exists a technique $\tau(\sigma) \in S(\sigma)$ whose "average" technique $\tilde{\tau}(\tau(\sigma))$ satisfies the condition: $\sum_{i\in N} a^i < d$.

Assumption 4.3. implies that for every $\sigma \in K$, there exists a technique $\boldsymbol{\tau}(\boldsymbol{\sigma}) \in S(\boldsymbol{\sigma})$ whose input-output matrix A of its "average" technique $\tilde{\tau}(\tau(\sigma))$ satisfies the condition: $\lambda(A) < 1$. Therefore, according to Fact 3 for every (non-negative) rate of profit in the non-degenerate interval $r \in \left[0, \frac{1-\boldsymbol{\lambda}(A)}{\boldsymbol{\lambda}(A)}\right)$ the "average" technique $\tilde{\boldsymbol{\tau}}(\boldsymbol{\sigma})$ has a unique positive wage rate and a unique positive price vector. Since we are interested only to non negative price vectors and distribution variables, from now on, for every $\sigma \in K$, we shall consider only techniques in the non-empty set $S_+(\sigma) = \left\{ \tau(\sigma) \in \overline{S}(\sigma) \middle| \tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix}, \lambda(A) < 1 \right\}$.

Proposition 4.1. below represents r-efficient techniques as limits of sequences of optimal techniques generated by solving iteratively Problems 3.1(i_f), with $i_f \in N_i$ and $i \in N$, as follows: given $r \geq 0$, $\sigma \in K$ and associated technology $T(\sigma)$, take an arbitrary technique $\boldsymbol{\tau}_0(\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}(0),..,a^{i_{F_i}}(0) \\ l_{i_1}(0),.....,l_{i_{F_i}}(0) \end{pmatrix}_{i\in N} \in S_+(\boldsymbol{\sigma})$ (set $S_+(\sigma)$ being non-empty because of Assumption 4.3.) and consider its "average" technique $\tilde{\tau}(\tau_0(\sigma)) = \begin{pmatrix} A_0 \\ l_0 \end{pmatrix}$ and its associated w_0 and p_0, where $p_0 \in \mathbb{R}^n_+$ and $w_0 > 0$. For each i_f, consider a solution $(a^{i_f}(1), l_{i_f}(1))^T \in \mathbb{R}^{n+1}_+$ to Problem 3.1(i_f) with respect to $\overline{p}_{r0} = ((1+r)p_0, w_0)$. An optimal technique $\boldsymbol{\tau}_1(p_0, w_0, r;\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}(1),...,a^{i_{F_i}}(1) \\ l_{i_1}(1),....,l_{i_{F_i}}(1) \end{pmatrix}_{i \in N}$ is therefore obtained with respect to the $\bar{p}_{r0}$, and let $\tilde{\tau}(\tau_1(p_0, w_0, r;\sigma)) = \begin{pmatrix} A_1 \\ l_1 \end{pmatrix} := \begin{bmatrix} a^1(1),\cdots,a^n(1) \\ l_1(1),......,l_n(1) \end{bmatrix}$ be the "average" technique associated to this technique. Its associated wage rates and price vector are w_1 and p_1 with $w_1 \geq w_0$, where the last inequality is a strict inequality if $\boldsymbol{\tau}_0(\boldsymbol{\sigma})$ is not an r-efficient technique, by Lemma 4.2.. Let $\begin{pmatrix} a^{i_f}(2) \\ l_{i_f}(2) \end{pmatrix} \in \mathbb{R}_+^{n+1}$ be a solution to Problem 3.1.(i_f) with respect to $\bar{p}_{r1} = ((1+r)p_1, w_1)$. A new optimal technique $\boldsymbol{\tau}_2(p_1, w_1, r;\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}(2),...,a^{i_{F_i}}(2) \\ l_{i_1}(2),.....,l_{i_{F_i}}(2) \end{pmatrix}_{i \in N}$ is therefore obtained with respect to the this extended price vector $\bar{p}_1$ and let $\tilde{\tau}_2(\tau_2(p_1, w_1, r;\sigma)) = \begin{pmatrix} A_2 \\ l_2 \end{pmatrix} := \begin{bmatrix} a^1(2),\cdots,a^n(2) \\ l_1(2),......,l_n(2) \end{bmatrix}$ be the "average" technique associated to this technique with associated w_2 and p_2 with $w_2 \geq w_1$. By iterating this process, a sequence $\{\tau_h(p_{h-1}, w_{h-1}, r;\sigma)\}_{h=0,1,2...} = \left\{ \begin{pmatrix} a^{i_1}(h),...,a^{i_{F_i}}(h) \\ l_{i_1}(h),.....,l_{i_{F_i}}(h) \end{pmatrix}_{i \in N} \right\} \subset S_+(\sigma)$ of optimal techniques is generated (where $\tau_0(p_{-1}, w_{-1}, r;\sigma) = \tau_0(\sigma)$). This sequence is called an *r-cost-minimizing sequence starting from* $\tau_0(\sigma)$. Notice that a sequence $\{w_h\}_{h=0,1,2,...}$ of wage rates and a sequence $\{p_h\}_{h=0,1,2,...}$ of price vectors are implicitly defined for any r-cost-minimizing sequence starting from $\tau_0(\sigma)$.

Proposition 4.1. *Let* $\boldsymbol{\tau}_0(\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}(0),...,a^{i_{F_i}}(0) \\ l_{i_1}(0),.....,l_{i_{F_i}}(0) \end{pmatrix}_{i \in N} \in S_+(\boldsymbol{\sigma})$ *be a technique and let* $r \in \left[0, \dfrac{1-\lambda(A_0)}{\lambda(A_0)}\right)$. *Let* $p_0 \geq 0$ *and* $w_0 > 0$ *the price vector and the wage rate associated with the "average" technique* $\tilde{\tau}_0(\tau(\sigma)) = \begin{pmatrix} A_0 \\ l_0 \end{pmatrix}$. *Then any r-cost-minimizing sequence starting from* $\boldsymbol{\tau}_0(\boldsymbol{\sigma})$ *has at least a convergent subsequence whose limit is an r-efficient technique.*

Proof. Denote by Φ the set of r-efficient techniques in $S_+(\sigma)$. For all h = 0,1,2,…., technique $\tau_h(\sigma)$ belongs to the compact set, $\overline{S}(\boldsymbol{\sigma}) = \prod_{i \in N}(\overline{S}^{i_1}(\boldsymbol{\sigma}^{i_1}) \times .... \times \overline{S}^{i_{F_i}}(\boldsymbol{\sigma}^{i_{F_i}}))$ where sets $\overline{S}^{i_f}(\sigma^{i_f})$ have been defined in the proof of Lemma 3.1. Define set $\overline{S}_+ := \left\{ \tau(\sigma) \in \overline{S}(\sigma) \mid \tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix}, \lambda(A) \leq \dfrac{1}{1+r} \right\}$ and the mapping: $\omega : \overline{S}_+ \to \overline{S}_+$ by the rule $\omega(\tau(\sigma)) = \left\{ \hat{\tau}(\sigma) = \begin{pmatrix} \hat{a}^{i_1}, \hat{a}^{i_2},..., \hat{a}^{i_{F_i}} \\ \hat{l}_{i_1}, \hat{l}_{i_2},....., \hat{l}_{i_{F_i}} \end{pmatrix}_{i \in N} \in \overline{S}_+(\sigma) \middle| \hat{\tau}(\sigma) = \tau(p, w, r, \sigma), \right.$ where p, w and r satisfy $(1+r)pA + wl = p$ and $\left. \tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix} \right\}$; i.e. mapping ω associates to each technique $\tau(\sigma)$ the set of techniques which are optimal at the price vector and wage rate generated by $\tilde{\tau}(\tau(\sigma))$. By definition, any cost-minimising sequence starting from $\tau_0(\sigma)$ satisfies the condition: $\tau_{h+1}(\sigma) \in \omega(\tau_h(\sigma))$ with h = 0,1,2,…… By Fact 3 and the continuity of the "aggregation" rule function $w : \overline{S}_+ \to \mathbb{R}_+$

defined by $w(\tau(\sigma)) = \dfrac{1}{1+l[I-(1+r)A]^{-1}d}$ for $\lambda(A) < \dfrac{1}{1+r}$ and $w(\tau(\sigma)) = 0$ for $\lambda(A) = \dfrac{1}{1+r}$ is continuous and, moreover, by Lemma 4.2.: $w(\tau_h(\sigma)) < w(\tau_{h+1}(\sigma))$ if $\tau_h(\sigma) \notin \Phi$ and $\tau_{h+1}(\sigma) \in \omega(\tau_h(\sigma))$. Finally, it is also possible to check that ω is an upper hemi continuous correspondence. Thus, every convergent subsequence of the cost-minimising sequence $\{\tau_h(\sigma)\}$ starting from $\tau_0(\sigma)$ has a limit in Φ (see Bazaraa, Sherali, Shetti (1993, p. 249)). ♦

Proposition 4.2. *Let* $\boldsymbol{\tau}_0(\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}(0), a^{i_2}(0), \ldots, a^{i_{F_i}}(0) \\ l_{i_1}(0), l_{i_2}(0), \ldots\ldots, l_{i_{F_i}}(0) \end{pmatrix} \in S_+(\boldsymbol{\sigma})$ *be a technique and* $r \in \left[0, \dfrac{1-\lambda(A_0)}{\lambda(A_0)}\right)$. *Then the wage rate sequence* $\{w_h\}_{h\in\mathbb{N}_0}$ *associated with any r-cost-minimizing sequence starting from* $\tau_0(\sigma)$ *converges to the same limit w*, where* $w^* = \max\left\{ w \in \mathbb{R} \,\middle|\, p = (1+r)pA + wl, p \in \mathbb{R}^n_+ \ , \ \tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix}, \tau(\sigma) \in S(\sigma) \right\}$. *Moreover, this limit is independent upon the initial technique* $\tau_0(\sigma)$.

Proof Let $\{\tau_h(p_{h-1}, w_{h-1}, r; \sigma)\}_{h=0,1,2,\ldots}$ be an r-cost-minimizing sequence starting from $\boldsymbol{\tau}_0(\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}(0), a^{i_2}(0), \ldots, a^{i_{F_i}}(0) \\ l_{i_1}(0), l_{i_2}(0), \ldots\ldots, l_{i_{F_i}}(0) \end{pmatrix}_{i\in N}$. By definition of r-cost-minimizing sequence, for every h, $p_h = (1+r)p_h A_h + w_h l_h \geq (1+r)p_h A_{h+1} + w_h l_{h+1}$. From Lemma 4.2, $w_h \leq w_{h+1}$, with strict inequality if $\tau_h(\sigma)$ is not optimal at prices p_h and wage rate w_h. Because of

Assumption 4.1., $p_1 + \ldots + p_n + w = 1$; hence $w \leq 1$, since $p \geq 0$. Therefore, sequence $\{w_h\}$ is monotonously non-decreasing and upper bounded, so it is convergent. Let w^* be its limit. Suppose now that there exist two initial techniques $\tau(\sigma)$, $\tau'(\sigma) \in S_+ (\sigma)$, with $\tau(\sigma) \neq \tau'(\sigma)$, whose r-cost minimising sequences starting from $\tau(\sigma)$ and $\tau'(\sigma)$ have, respectively, w^* and $w^{*\prime}$ as limit wage rates, and suppose that $w^* \neq w^{*\prime}$. Without loss of generality we can assume that $w^* < w^{*\prime}$. Hence, by standard results (see, for example, Kurz and Salvadori (1998), Bidard (2004)), it can be shown that there must be an industry index i such that $(1+r)p^* a^{i\prime} + w^* l_i' < (1+r)p^* a^i + w^* l_i$ which contradicts Proposition 4.1, by exploiting the properties of the "aggregation rules". Hence, $w^* = w^{*\prime}$. A similar argument proves the last assertion.♦

The *unique* wage rate w^* yielded by all r-efficient techniques associated with a capability profile $\sigma \in K$ is called the *r-efficient wage rate of capability profile σ.*

5. Dynamic capabilities and the evolution of technology

Two facts widely accepted by the economic literature are that firms have limited knowledge of the set of all possible technologies and, consequently, that they behave adaptively (see, e.g., Nelson and Winter (1982)), and that firms' capabilities evolve incessantly over time (see, for example, Teece, Pisano and Shuen (1995)). In this section we shall extend the static model developed in the previous sections to allow for adaptive changes in industry technologies as driven by the evolution of firms' capabilities. As already said, the Appendix deals with the case of evolution of firms' techniques. We obtain a model describing an economy in which dynamic capabilities generate a ceaseless change in technologies and in their associated *r*-efficient techniques, price vectors and wages. Given that *r*-efficient techniques are actually long period techniques at rate of profit *r* (see footnote 12), our dynamic process has a strong marshallian flavour in terms of his distinction between secular and long period configurations (see Marshall (1890, p. 315). For a recent restatement of this view, see Arthur (2009, p. 200)).

In addition to the assumptions already made, in this section we shall adopt two additional technical assumptions:

Assumption 5.1. For every $i_f \in F_i$ and $i \in N$, set K^{i_f} is compact; moreover, for every $j \in N_+$ the mapping $b_{kj}^{i_f} : K \to \mathbb{R}_+^m$ defined by the vectors of extracted characteristics associated to each knowledge profile are continuous functions.

Assumption 5.2. There exists a positive number δ such that, for every $\sigma \in K$ and every $i_f \in F_i$ and $i \in N$, the non-zero elements of matrices $B^{i_f}(\boldsymbol{\sigma}^{i_f})$ are not lower than δ.

Recall that for any capability profile $\sigma \in K$, we have an associate technology $T(\boldsymbol{\sigma}) = \left(B^{i_f}(\boldsymbol{\sigma})\right)_{\substack{i_f \in N_i \\ i \in N}} = \left(\left(b_j^{i_f}(\boldsymbol{\sigma})\right)_{j \in N_+}\right)_{\substack{i_f \in N_i \\ i \in N}}$. By Assumption 5.1., the set $Q = \left\{T(\sigma) \middle| \sigma \in K\right\} \subset \mathbb{R}_+^{m \times \mathfrak{N}}$ of possible technologies (i.e. the set of possible extraction characteristic matrices in all industries generated by all possible states of firms' capabilities) is a compact set. The minimum β of non-zero elements of every matrix $B^{i_f}(\boldsymbol{\sigma})$ is positive for all $\sigma \in K$ by Assumption 5.2.. On the other hand, let γ be the maximum of the elements of the matrix (c^1, ..., c^n). For $\sigma \in K$ set $\bar{S}(\sigma) = S(\sigma) \cap \Delta$, where $\Delta := \left\{(x_{ji_f}) \in \mathbb{R}_+^{(n+1)\times \mathfrak{N}} \mid x_{ji_f} \leq \frac{2\gamma}{\beta},\, j \in N_+,\, i_f \in F_i,\, i \in N\right\}$. Obviously, $\bar{S}(\sigma)$ is compact and non-empty for all $\sigma \in K$.[11]

Lemma 5.1. *Correspondence* $\bar{S} : Q \to \Delta$ *is continuous.*

Proof. Let $\{\sigma^t\}_{t=0,1,2,...}$ be an arbitrary sequence of capability profiles in K converging to $\sigma \in K$. By continuity $\lim_{t\to\infty} B^{i_f}(\sigma^t) = B^{i_f}(\sigma)$ for every $i_f \in N_i$, and $\{\tau^t\}_{t=0,1,2..}$ be an arbitrary sequence of techniques with $\boldsymbol{\tau}^t = (x^{i_1 t}, ..., x^{i_{F_i} t})_{i \in N} \in \bar{S}(\boldsymbol{\sigma}^t)$, converging to technique $\boldsymbol{\tau} = (x^{i_1}, ..., x^{i_{F_i}})_{i \in N}$. Then, $B^{i_f}(\sigma^t) \cdot x^{i_f t} \geq c^i$ and $x^{i_f t} \in \Delta$ for every $i_f \in N_i$, $i \in N$ and t. Therefore, $B^{i_f}(\sigma) \cdot x^{i_f} \geq c^i$ and $x^{i_f} \in \Delta$ for every $i_f \in N_i$ and $i \in N$. It follows

11 Non emptiness follows from the fact that for every $\sigma \in K$, every $i_f \in N_i$, $i = N$ and $j \in N_+$: $\frac{2\gamma}{\beta} d \in S^{i_f}(\sigma)$ and $\hat{b}_j^{i_f}(\sigma) \frac{2\gamma}{\beta} d > c_{ji}$ if $c_{ji} > 0$, where $\hat{b}_j^{i_f}(\sigma)$ is the j-th row of matrix $B^{i_f}(\sigma)$.

that $\tau \in \bar{S}(\sigma)$, hence $\bar{S}$ is an upper hemi-continuous.

Next, suppose that $\bar{S}$ is not lower hemi-continuous. Then, for some $\sigma \in K$, some $\tau \in \bar{S}(\sigma)$ and some $\varepsilon > 0$, there exists a sequence $\{\sigma^t\}_{t=0,1,2,..}$ in K so that $\lim_{t\to\infty} \sigma^t = \sigma$ and $\bar{S}(\sigma^t) \cap U_\Delta(\tau,\varepsilon) = \varnothing$ for all t where $U_\Delta(\tau,\varepsilon) := \{\tau' \in \Delta \,|\, |\tau' - \tau| < \varepsilon\}$, but this is not possible. As a matter of fact, let $\tau' = ((x'_{ji_f})_{\substack{j\in N_+ \\ i_f \in N_i}})_{i\in N}$ be a matrix in $\mathbb{R}_+^{(n+1)\times\mathfrak{N}}$ so that

$x'_{ji_f} := \min\left(x_{ij} + \sqrt{\frac{1}{2}\frac{\varepsilon}{(n+1)\mathfrak{N}}}, \frac{2\gamma}{\beta} \right)$. Then, $\tau' \in U_\Delta(\tau,\varepsilon)$, and for every $i_f \in N_i$, $i \in N$ and $j \in N_+$, $\hat{b}^i_j(\sigma)(x'_{ji_f})_{\substack{j\in N_+ \\ i_f \in N_i}} > c_{ji}$ if $c_{ji} > 0$, where $\hat{b}^{i_f}_j(\sigma)$ is the j-th row of $B^{i_f}(\sigma)$. Then, by continuity, for every small enough $\varepsilon' > 0$, for all $\sigma' \in U_K(\sigma,\varepsilon')$ and for every $i_f \in N_i$ and $i \in N$, we have $\hat{b}^i_j(\sigma')(x'_{ji_f})_{\substack{j\in N_+ \\ i_f \in N_i}} > c_{ji}$ if $c_{ij} > 0$. This implies $B^{i_f}(\sigma')(x'_{ji_f})_{\substack{j\in N_+ \\ i_f \in N_i}} \geq c^i$ for every $i_f \in N_i$ and $i \in N$, and therefore, $\tau' \in \bar{S}(\sigma')$. Then, for t large enough, $\tau' \in U_\Delta(\tau,\varepsilon)$ and $\tau' \in \bar{S}(\sigma^t)$. This contradicts that $\bar{S}(\sigma^t) \cap U_\Delta(\tau,\varepsilon) = \varnothing$ for all t. ♦

Let $\lambda_m(\sigma)$ be the minimum of Frobenius roots of feasible "average" techniques associated with the capability profile $\sigma \in K$, i.e. $\lambda_m(\sigma) = \min\left\{ \lambda(A) \in \mathbb{R}_+ \,|\, \tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix}, \tau(\sigma) \in \bar{S}(\sigma) \right\}$. Notice that $\lambda_m(\sigma)$ is well defined because $\bar{S}(\sigma)$ is compact by Assumption 5.1. and function $\lambda : \mathbb{R}_+^{n\times n} \to \mathbb{R}_+$ is continuous.

Lemma 5.2. *Function* $\lambda_m : K \to \mathbb{R}_+$ *is continuous.*

Proof Immediate from Lemma 5.1, from the continuity of λ and from Berge's maximum value theorem. ♦

Define Λ by $\Lambda = \max\{\lambda_m(\sigma) \mid \sigma \in K\}$. Λ is well defined because *K* is compact and λ_m is continuous by the previous lemma. Define Π by $\Pi = \frac{1-\Lambda}{\Lambda}$ for Λ ≠ 0 and Π= ∞ for Λ = 0. Λ < 1 holds obviously from Assumption 4.3, and therefore Π > 0. If *r* < Π holds, there is some technique $\tau(\sigma) \in \bar{S}(\sigma)$ for all $\sigma \in K$ so that $r < \Pi = \frac{1-\Lambda}{\Lambda} \leq \frac{1-\lambda_m(\sigma)}{\lambda_m(\sigma)} = \frac{1-\lambda(A)}{\lambda(A)}$, where $\tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A \\ l \end{pmatrix}$.

By Proposition 4.2., the *r*-efficient wage rate associated to the capability profile $\sigma \in K$ is uniquely determined, hence for every $r \in (0, \Pi)$, the mapping $w_r : K \to \mathbb{R}_+$ associating the *r*-efficient wage rate $w_r(\sigma)$ to each capability profile $\sigma \in K$ is a well defined function.

Lemma 5.3. *Function* $w_r : K \to \mathbb{R}_+$ *is continuous.*

Proof. Let $\sigma \in K$, then, as usual, we have an associated technique $\boldsymbol{\tau}(\boldsymbol{\sigma}) = \begin{pmatrix} a^{i_1}, a^{i_2}, \ldots, a^{i_{F_i}} \\ l_{i_1}, l_{i_2}, \ldots\ldots, l_{i_{F_i}} \end{pmatrix}_{i \in N}$ and the associated "average" technique $\tilde{\tau}(\tau(\sigma)) = \begin{pmatrix} A(\sigma) \\ l(\sigma) \end{pmatrix}$ with $A(\sigma) \in \mathbb{R}_+^{n \times n}$ and $l(\sigma) \in \mathbb{R}_+^n$. Define for each $r \in (0, \Pi)$ a function $w(r, \cdot) : K \to \mathbb{R}_+$ as follows: $w(r, \sigma) = \frac{1}{l(\sigma)[I-(1+r)A(\sigma)]^{-1} d + 1}$ for

$\lambda(A(\sigma)) < \frac{1}{1+r}$, and $w(r;\sigma) = 0$ for $\lambda(A(\sigma)) \geq \frac{1}{1+r}$. Then, by the continuity of the "aggregation" rule and the mapping associating to each capability profile σ a technology $T(\sigma)$, for every $r \in (0, \Pi)$, $w(r;\cdot)$ is continuous on K. Define the function $\hat{w}_r : K \to \mathbb{R}_+$ by the following rule $\hat{w}_r(\sigma) = \max_{\tau \in \bar{S}(\sigma)} w(r,\tau) = \max\left\{ w \geq 0 \mid p = (1+r)pA(\sigma) + wl(\sigma), p \geq 0, \sum_{i=1}^{n} p_i + w = 1 \right.$ and $\left. \tilde{\tau}(\sigma) = \begin{pmatrix} A(\sigma) \\ l(\sigma) \end{pmatrix}, \tau(\sigma) \in \bar{S}(\sigma) \right\}$. Since correspondence $\bar{S}$ and function w are continuous, function $\hat{w}_r$ is continuous because of Berge's maximum value theorem. Since an r-efficient technique exists in $\bar{S}(\sigma)$ and any r-efficient technique has a common wagre rate equal to the r-efficient wage rate, and since $w(r,\tau) \geq w(r,\tau')$ whenever $\tau \in \bar{S}(\sigma)$ and $\tau' \in S(\sigma) \setminus \bar{S}(\sigma)$, then $\hat{w}_r(\sigma) = \max_{\tau \in \bar{S}(q)} w(r,\tau) =$ $w_r(\sigma) =$ $\max_{\tau \in \bar{S}_+(q)} w(r,\tau)$. ♦

The extraction of characteristics is envisaged to depend on time-dependent variables such as experience, knowledge etc. (see Dosi and Grazzi (2006)). Thus, as already pointed out, it is natural to allow for changes of technology over time driven by the evolution of capabilities. We maintain that the evolution of capabilities is determined endogenously through a process of adaptive dynamics, which is formalized as follows (see also D'Agata (2005, 2010)): At time 0, a capability profile σ_0 is given and an r-efficient technique τ_0 with wage rate $w_r(\sigma_0)$ is associated to it for some non-negative profit rate r. By using capability profile σ_0 in period 0, firms expand their knowledge and skills (i.e. they acquire new capabilities) ``around'' the current profile σ_0. At time 1, firms have a set of feasible capabilities $\psi(\sigma_0)$ available and in that period they choose the optimal capability profile σ_1, which is the one whose associated r-efficient wage rate $w_r(\sigma_1)$ is maximum with respect to the r-efficient wage rates of all feasible capabilities profiles. And so on. As time passes, we obtain a sequence of sets of capability profiles discovered each time and an associated sequence of r-efficient techniques and prices and wage rates. For any initial capability profile, we will show the existence of a converging subsequence of capability profiles and of associate r-efficient techniques and wage rates such that the limit capability profiles can be considered Marshallian (local) “secular equilibria”. Let $\psi : K \to K$ be the *capability evolution correspondence*; if σ_t is the capability profiles chosen at time t, set $\psi(\sigma_t)$ is interpreted as the set of capability profiles discovered at that time and which are available for production activities at time t+1. This correspondence should catch the idea that firms have not complete knowledge of the whole knowledge set, and that new capabilities (and associated technologies and techniques) are discovered over time. This also implies that knowledge growth and technical change are eminently local in character (see Atkinson and Stiglitz (1969), Antonelli (1995)).

Assumption 5.3. The capability evolution correspondence ψ is compact valued and lower hemi-continuous, with $\sigma \in \psi(\sigma)$ for all $\sigma \in K$.

The last condition in Assumption 5.3. is not essential for our results. It is adopted for its obvious and reasonable economic meaning. Let $r \geq 0$ and let σ^0 be the initial technology, and $\tau_r(\sigma^0)$ and $w_r(\sigma^0)$ be respectively an r-efficient technique and the (unique) r-efficient wage rate associated with the capability profile σ^0. An adaptive process $\{\sigma^t\}_{t=0,1,2,...}$ starting from σ^0 is defined by the rule: $\sigma^{t+1} := \arg\max_{\sigma \in \psi(\sigma^t)} w_r(\sigma)$ for $t = 0,1,2....$ For some $r \geq 0$, a capability profile $\sigma \in K$ is an *r-local secular equilibrium* (*r-LSE*) if $w_r(\sigma) = \max_{\sigma' \in \psi(\sigma)} w_r(\sigma')$. As already said, the concept of r-LSE is very close to Marshall's concept of "secular equilibrium".

The adaptive process is well-defined as function w_r is continuous by Lemma 5.2, and the capability evolution correspondence ψ is compact valued, by Assumption 5.3.

Lemma 5.4. *For any initial capability profile $\sigma \in K$ and for every $r \in (0, \Pi)$, any adaptive process starting from σ has a convergent subsequence. Moreover, the sequence of the associated r-efficient wage rates, $\{w_r(\sigma^t)\}_{t=0,1,2...}$, converges as $t \to \infty$.*

***Proof*.** The first part is an immediate consequence of the Bolzano-Weierstrass property of metric spaces. As for the last part, from Assumption 5.3, $\sigma^t \in \psi(\sigma^t)$ so that $w_r(\sigma^t) \leq w_r(\sigma^{t+1})$ for all $t \geq 0$. Because of Assumption 4.1. and $p \geq 0$, $w \leq 1$. Then, since the sequence $\{w_r(\sigma^t)\}_{t=0,1,2...}$ is monotonously non-decreasing and upper bounded, it converges.♦

Proposition 5.1. *For every* $\sigma_0 \in K$ *and for every* $r \in (0, \Pi)$*, the limit of every adaptive process starting from* σ_0 *is an r-LSE.*

Proof Let $\sigma_0 \in K$ the initial technology and $r \in (0, \Pi)$. Let $\{\sigma^t\}_{t=0,1,2..}$ be a convergent subsequence of an adaptive process starting from σ_0, and with limit σ. We assume counterfactually that σ is not an *r*-LSE. Then, there exists a capability profile $\sigma' \in \psi(\sigma)$ so that $w_r(\sigma) < w_r(\sigma')$. Hence, $H(\boldsymbol{\alpha}) \cap \boldsymbol{\psi}(\boldsymbol{\sigma}) \neq \varnothing$ where $H(\boldsymbol{\alpha}) = \left\{\boldsymbol{\sigma} \in K \,\middle|\, w_r(\boldsymbol{\sigma}) > \boldsymbol{\alpha}\right\}$ and $w_r(\sigma) < \alpha < w_r(\sigma')$. By lower hemi continuity, for *t* "big enough" $H(\boldsymbol{\alpha}) \cap \boldsymbol{\psi}(\boldsymbol{\sigma}^t) \neq \varnothing$ which contradicts the definition of σ^t, because $w_r(\sigma^t) \leq w_r(\sigma)$ by what has been said in the proof of Lemma 5.4.. ♦

Finally, it may be useful emphasise again that also the dynamic analysis has been carried out for a given rate of profit and increasing wage rates. The choice of the rate of profit as exogenous distributive variable, as it is well known by the literature on linear production models (see Kurz and Salvadori (1998) and Bidard (2004)) is only a matter of analytical convenience, and the same results could have been obtained by giving the wage rate exogenously.

6. Final remarks

In this paper we have developed a characteristic-based model for the endogenous determination of technical coefficients in a linear economy. Our model provides a rigorous analysis of endogenous determination of technical coefficients and associated production prices and distribution by using an approach which is easily interpretable in terms of the "procedural" approach to technology. In this sense, it can be interpreted as an attempt to fill in the gap, emphasized by Dosi and Grazzi (2006), between the procedure-centered and the input-output-centered representation of technology, by formally developing the characteristic-based view proposed by von Tunzelmann (2003). A simple dynamic analysis is also provided, which provides a formalization of the adaptive evolution of technology and techniques over time driven by the evolution of firms' productive capabilities.

We have explicitly allowed for heterogeneity of firms in terms of capabilities in line with the literature (see, e.g. Dosi (1982, 2000), Metcalfe (2010), Dosi and Nelson (2010)). This makes our model a theoretical foundation to Dosi and Grazzi's work on the distribution and evolution of technical coefficients in linear economies (Dosi and Grazzi (2006)). Heterogeneity of firms is endogenised in the Appendix by replacing the second part of Assumption 3.1. with the assumption that firms discover the set of feasible techniques over time according to the same model of adaptive dynamics used in Section 5. As emphasised in other works (see D'Agata (2005, 2010)) this adaptive dynamics can additionally deal with widely recognised phenomena like lock-in and dynamic inefficiencies. D'Agata (2010) shows also that our model of technical change is able to deal with changes in the number of produced goods.

The description of technology and knowledge here proposed has been deliberately kept here at the most simplest possible level. As it has just been said, it is possible in principle to remove the assumption that firms have a complete knowledge of the set of feasible techniques (Assumption 3.1.). It is possible also to develop a more complex description of technology by removing the assumption of linearity in the extraction technology (Assumption 2.2.) and by removing the assumption of a unique vector c^i of minimum characteristics in each industry (Assumption 2.1.) or by replacing it by a more general description of minimum requirements for production. These generalisations are not done here as they can fruitfully be carried out only with reference to specific issues in production theory. The same remark holds true for the formalisation of knowledge which in this paper has been done in a very simple way. Clearly, these formalisations are worth of future attention.

APPENDIX

In this appendix we endogenise the heterogeneity of firms by weakening the last part of Assumption 3.1., i.e. by allowing firms to know only a subset of their set of feasible techniques and to adaptively adopt techniques as their knowledge of the feasible set of techniques changes over time. The model will be developed in a sketchy way as, from the formal point of view, it is similar to the one developed in Section 5. Since the model can be interpreted as a "short-period" version of the dynamics considered in that section, the dynamics here considered should be interpreted as a "fast" dynamics occurring within each period considered in Section 5. This means that in what follows the capability profile of the economy $\sigma \in K$ and the extended price vector $p_r=((1+r)p_1, (1+r)p_2, \ldots, (1+r)p_n, w)$ will be here considered given.

Give $\sigma \in K$, the set of feasible techniques $S^{i_f}(\sigma)$ is therefore given as well. Like the dynamics in Section 5, time is discrete (with the qualification previously given) and we assume that whenever firm i_f is using a technique x, then, during the relevant period it "discovers" set $\xi^{i_f}(x) \subset S^{i_f}(\sigma)$. Mapping $\boldsymbol{\xi}^{i_f} : S^{i_f}(\boldsymbol{\sigma}) \to S^{i_f}(\boldsymbol{\sigma})$ is assumed to be lower hemi continuous.

If at time t-1 firm i_f is using technique $x_{t-1} \in S^{i_f}(\sigma)$, then Problem 3.1.($i_f$) becomes:

Problem A(i_f): min $p_r \cdot x$ s.t. $x_t \in \xi^{i_f}(x_{t-1})$.

Given any initial technique $x_0^{i_f} \in S^{i_f}(\sigma)$ and assuming a strictly positive vector p_r, it follows that any solution to Problem A(i_f) must belong to a compact set $\overline{S}^{i_f}(\boldsymbol{\sigma}) \subset \mathbb{R}_+^{n+1}$ (see also the proof of Lemma 3.1.). Thus, from Problem A(i_f) we obtain a sequence $\left\{x_t^{i_f}\right\}_{t=0,1,2,...}$ of techniques in $\overline{S}^{i_f}(\boldsymbol{\sigma})$. By using the argument in Section 5, it is possible to show that this sequence converges to some technique $x^{i_f} = \begin{pmatrix} a^{i_f} \\ l_{i_f} \end{pmatrix} \in \overline{S}^{i_f}(\boldsymbol{\sigma})$ so that $p_r \cdot x^{i_f} \leq p_r \cdot x'$ for every $x' \in \boldsymbol{\xi}^{i_f}(x^{i_f})$; i.e. technique $x^{i_f} = (a^{i_f}, l_{i_f})$ is a local optimum. Given that each firm may start from a different initial technique $x_0^{i_f}$ and/or may have a different "exploration" maps $\boldsymbol{\xi}^{i_f}$, it is possible that the optimal techniques $x^{i_f} = (a^{i_f}, l_{i_f})$ differ from one firm to another firm in the same industry and may not be an optimal technique in the sense of Problem 3.1.(i_f).

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