High risk or low cost – Dichotomous choices of R&D strategy by startups in markets for technology

Joachim Henkel  
Technical University of Munich  
TUM School of Management  
henkel@wi.tum.de  

Thomas Roende  
Copenhagen Business School  
Department of Innovation and Organizational Economics  
thr.ino@cbs.dk

Abstract
Technology-focused acquisitions of startups by incumbents are highly diverse, comprising radical but also incremental innovations. We build a model of R&D competition between an incumbent and a startup to explain this diversity. Each player chooses investment level and success probability, or risk, of its project. After outcome realization, the incumbent commercializes the most valuable project, and, where necessary, acquires the startup to this end. We find that two locally optimal strategies for the startup can exist, characterized respectively by higher risk or lower cost than the incumbent’s equilibrium strategy. Depending on the R&D technology, either of them may be the startup’s globally optimal strategy. With two startups, numerical analysis confirms this finding. The intuition behind our results is that a startup pursuing a high risk strategy aims at technological superiority, while one following a low cost strategy banks on having the only successful R&D project. Our model thus suggests a dichotomy of technology-focused acquisitions of startups by incumbents.
Abstract: Technology-focused acquisitions of startups by incumbents are highly diverse, comprising radical but also incremental innovations. We build a model of R&D competition between an incumbent and a startup to explain this diversity. Each player chooses investment level and success probability, or risk, of its project. After outcome realization, the incumbent commercializes the most valuable project, and, where necessary, acquires the startup to this end. We find that two locally optimal strategies for the startup can exist, characterized respectively by higher risk or lower cost than the incumbent’s equilibrium strategy. Depending on the R&D technology, either of them may be the startup’s globally optimal strategy. With two startups, numerical analysis confirms this finding. The intuition behind our results is that a startup pursuing a high risk strategy aims at technological superiority, while one following a low cost strategy banks on having the only successful R&D project. Our model thus suggests a dichotomy of technology-focused acquisitions of startups by incumbents.
1. Introduction

Research in the fields of innovation and entrepreneurship has established that young firms tend to be better at radical innovation than incumbents (e.g., Arrow 1962, Reinganum 1983, Scherer and Ross 1990, van Praag and Versloot 2007) and that established firms frequently source such innovations through acquisitions (e.g., Baumol 2010, Bloningen and Taylor 2000, Granstrand and Sjölander 1990, Hall 1990, Hsu 2006, Lerner and Merges 1998). A case in point is Pharmasset, Inc., acquired by biotech firm Gilead in 2011 for its novel therapeutic concept against Hepatitis C virus infections (Bloomberg, 2011). The acquisition price of USD 11 billion suggests that the innovation central to the transaction was a radical one, significantly superior to Gilead’s own solution. Yet, as already Schumpeter (1942) noted, important innovations may also come from incumbents due to their superior capital position; and in turn, not all innovations generated by startups are radical. Young firms may also be founded, and acquired, on the basis of relatively minor innovations. Consider the case of iLytix Systems AS, bought in 2005 by the software maker SAP for an undisclosed sum. The startup’s key technology was its product XL Reporter, a tool that establishes an interface between SAP’s software and Microsoft Excel (Computer Business Review, 2005). This technology was clearly not a radical innovation; still, the product allowed SAP to swiftly satisfy user needs for which it had no existing internal solution.

These examples raise the question of whether Gilead acquiring Pharmasset and SAP acquiring iLytix are just two extremes on a continuum or if there is a more fundamental difference. In both cases, an incumbent itself active in R&D acquired a young firm for its technology. The acquisitions differed, however, with respect to the radicalness of the innovation, the transaction volume, and the fact that Gilead had its own solution while SAP did not. Thus, we ask, how do R&D competition between incumbents and entrants and the prospect of an acquisition shape the firms’ R&D strategies. Specifically, how do these considerations affect the drivers of innovation radicalness, that is, risk level and investment?

We are grateful to Jan Paul Stein for pointing us to this example.
Existing research offers a number of answers to these questions. Gans and Stern (2000) are the first to theoretically analyze R&D competition between a startup and an incumbent where acquisition of the former is an option. The authors find that the parties’ incentives to invest depend strongly on their respective bargaining power in licensing negotiations, which, in turn, depends on the strength of intellectual property rights, the incumbent’s threat to develop its own solution, and the young firm’s possibilities of product market entry. Kleer and Wagner (2013) assume that small firms are more efficient in R&D, and large firms are better in exploitation. As a result, small firms invest more in R&D. With sufficiently large differences in efficiency, large firms refrain from performing R&D and rely exclusively on acquisitions. In a similar vein, Phillips and Zhdanov (2012) build a model where firms are heterogeneous with respect to production costs, and show that large firms with low costs may refrain from doing R&D and instead acquire smaller R&D active competitors. Henkel et al. (2015) model R&D competition between an incumbent and an arbitrary number of startups, assuming a startup needs to be acquired by the incumbent for its invention to be commercialized. Firms choose the success probability of their respective R&D project, rather than investment as in the models cited above. The authors identify an equilibrium in which the incumbent chooses the safest project, and startups pursue pairwise different projects that become riskier the more firms participate in the competition.

While existing studies greatly enhance our understanding of R&D competition between entrants and incumbents when acquisition is an option, they restrict the firms’ choice of R&D strategies to one dimension, either the investment level or the success probability. While legitimate for simplification, such a restriction excludes interactions and trade-offs between the two dimensions—relationships that are likely crucial. For instance, it appears plausible that in order to produce radical innovations startups would pursue riskier R&D approaches, while incumbents might instead dedicate more resources to a project. We

Also Färnstrand Damsgaard et al. (2017) model R&D competition between an incumbent and an entrant using success probability, or risk level, as the players’ strategy variable. However, their model excludes the possibility of an acquisition.
thus extend existing research by studying both risk level and investment as elements of the players’ strategies.

To this end, we develop a game-theoretic model of R&D competition between one or two startups and one incumbent in which firms simultaneously choose success probability and investment level. Nature decides which projects succeed, upon which the incumbent realizes the most valuable project. If this project is owned by a startup, the incumbent acquires this firm (or its technology). We study a number of extensions of the model to demonstrate its robustness.

Our results point to a dichotomous choice of R&D strategy by high-tech startups that face R&D competition with incumbents and aim to be acquired. Under certain conditions, two locally optimal strategy choices for the startup exist, which we characterize as “high risk” and “low cost,” respectively. Pursuing the former, the startup chooses a lower success probability than the incumbent, with concomitant higher value in case of success. In contrast, the low cost strategy implies a lower investment level. Depending on the R&D technology, either of the two strategies may be the startup’s globally optimal strategy. With two startups, numerical analysis shows that either both go for a high risk strategy, both go for a low cost strategy, or each picks a different type of strategy. Our findings are robust to variations in the players’ relative bargaining powers.

The intuition behind our results is that a startup pursuing a high risk strategy aims at technological superiority, while one that follows a low cost strategy banks on having the only successful R&D project. These different situations affect the marginal benefits of higher risk and higher investment differentially, which gives rise to the dichotomy of locally optimal strategies. Our model thus suggests a structural dichotomy rather than a continuous diversity of technology-focused acquisitions of startups by incumbents. In addition, our analysis shows that the project with the highest value in case of success may originate either from the startup pursuing a riskier project, or from the incumbent investing more.

This paper contributes to three streams of literature. The first is the aforementioned literature on R&D competition between startups and incumbents when acquisition of the former is an option (Gans and Stern 2000, Henkel et al. 2015, Kleer and Wagner 2013, Phillips and Zhdanov 2012). Our research introduces a
dichotomy of the startups’ R&D strategies and hence of the acquisitions, thus adding clarity to the strong heterogeneity of technology-focused acquisitions.

Second, we contribute to research on markets for technology, which are growing fast and have become sizeable during the last decades (Arora et al. 2001, Athreye and Cantwell 2007, Robbins 2006). Sellers in these markets are both large companies looking for ways to improve revenues and small startups with limited possibilities to commercialize the technologies themselves. On the side of buyers, incumbent firms frequently acquire the technologies of small, innovative startups in order to stay ahead and to preempt competition (Bloningen and Taylor 2000, Grimpe and Hussinger 2008, 2009, Hall 1990, Lehto and Lehtoranta 2006, Lerner and Merges 1998,). Our study provides a more fine-grained understanding of technology-focused acquisitions by incumbents, explaining why minor technologies may also be subject to such acquisitions.

Third, our work relates to the large literature on R&D competition. Previous work has shown that the R&D strategy of a firm has many dimensions: the amount of resources to invest in R&D (Arrow 1962, Gilbert and Newbery 1982, Reinganum 1983), the composition of the R&D portfolio (Cabral 1994), the R&D trajectory to follow (Cardon and Sasaki 1998), the choice of risk and return (Anderson and Cabral 2007, Bhattacharya and Mookherjee 1986), and the correlation between own and competitors’ R&D outcomes (Dasgupta and Maskin 1987). However, researchers typically address these dimensions in isolation, ignoring possible interactions between them. In this paper, we contribute to the literature by considering a setup where firms choose the R&D investment and the risk of the project, the two most prominent R&D choices in the literature. We show that the choices are interdependent and that the entrant chooses either low investment and low risk or high investment and high risk.

---

3 Ali et al. (1993) consider the choice between a radical and an incremental project where the two types of projects involve different risk and investment. However, as the choice is discrete, the possibilities to study the interaction between risk and investment are limited.
In the next section, we introduce our model, which we analyze in Section 3. Section 4 addresses extensions of the model and robustness checks. The final section concludes. Most proofs appear in the Appendix.

2. Model Set-Up

The basic model has two risk-neutral players, the incumbent \((I)\) and the startup \((S)\). They play a three-stage game. In Stage 1, each player \(X (X = I, S)\) chooses the success probability, \(p_X\), of its innovation project, and the investment level or cost, \(c_X\). In Stage 2, Nature determines success or failure of the projects. With probability \(p_X\), the project of player \(X\) takes on the value \(\pi(p_X, c_X)\); with probability \((1 - p_X)\), it takes on a value of zero. We call \(\pi(\cdot, \cdot)\) the “value function”; it describes the value of an innovation project if it is successful and subsequently commercialized by the incumbent.\(^4\) We will sometimes refer to the “value in case of success” as VICOS. In Stage 3, if the project with the highest realized value is that of the startup, the incumbent acquires the firm (or its innovation) at a price equal to the difference in realized value between the startup’s and its own project. That is, regarding the negotiation, we assume that \(S\) has all the bargaining power and makes a binding take-it-or-leave-it offer to \(I\). We will relax this assumption in an extension to the model. If the realized value of the startup’s project is not greater than that of \(I\)’s, then the incumbent makes no acquisition and realizes its own project (or none, if both have failed). We make the following assumptions regarding the value function:

1) \[\pi: [0; 1] \times \mathbb{R}^+ \rightarrow \mathbb{R}, (p, c) \mapsto \pi(p, c).\]

2) \(\pi(p, c)\) is continuous and twice continuously differentiable.

3) \(\pi(p, c) \geq 0\).

4) \(\frac{\partial \pi}{\partial p} < 0\): a “safer” project yields lower value in case of success.

5) \(\frac{\partial \pi}{\partial c} > 0\): higher investment leads to higher value in case of success.

\(^4\) A “value function” is not to be confused with the term as used in dynamic programming.
6) The Hessian matrix of \( p\pi(p, c) \) is negative definite. This assumption implies concavity with respect to \( p \) and \( c \).

7) For given \( c \), the function \( p\pi(p, c) \) takes on its maximum at \( \bar{p}(c) \) in the interior of the interval \([0; 1]\).

8) For given \( p \), the function \( p\pi(p, c) - c \) takes on its maximum at \( \bar{c}(p) > 0 \).

Expected profits of the incumbent and the startup, respectively, are then given by

\[
\Pi_I(p_I, c_I, p_S, c_S) = p_I \pi(p_I, c_I) - c_I, \tag{1}
\]

\[
\Pi_S(p_I, c_I, p_S, c_S) = \begin{cases}
(1 - p_I) p_S \pi(p_S, c_S) - c_S & : p_S, c_S \geq \pi(p_I, c_I) \\
p_S \pi(p_S, c_S) - c_S & : p_S, c_S < \pi(p_I, c_I)
\end{cases} \tag{2}
\]

From the assumptions, it follows that the function \( p\pi(p, c) - c \), which is also the profit function of a monopolist not threatened by entry, takes on its global maximum at some point \((\bar{p}, \bar{c})\) with \( 0 < \bar{p} < 1 \) and \( \bar{c} > 0 \). It follows that the incumbent has a dominant strategy given by \((p_I^*, c_I^*) = (\bar{p}, \bar{c})\), and the value of its project in case of success equals \( \pi(\bar{p}, \bar{c}) \).

3. Solving the Model

3.1. First-Order Conditions and the Startup’s Best Responses

Partial differentiation of equations (1) and (2) yields the first-order conditions:

\[
\frac{\partial \Pi_I}{\partial p_I} \equiv p_I \pi(p_I, c_I) + p_I \frac{\partial \pi(p_I, c_I)}{\partial p_I} = 0 \tag{3}
\]

\[
\frac{\partial \Pi_I}{\partial c_I} \equiv p_I \frac{\partial \pi(p_I, c_I)}{\partial c_I} - 1 = 0 \tag{4}
\]

\[
\frac{\partial \Pi_S}{\partial p_S} \equiv \begin{cases}
\pi(p_S, c_S) + p_S \frac{\partial \pi(p_S, c_S)}{\partial p_S} - p_I \pi(p_I, c_I) & : p_S, c_S \geq \pi(p_I, c_I) \\
(1 - p_I) \left( \pi(p_S, c_S) + p_S \frac{\partial \pi(p_S, c_S)}{\partial p_S} \right) & : p_S, c_S < \pi(p_I, c_I)
\end{cases} = 0 \tag{5}
\]
\[
\frac{\partial n_S}{\partial c_S} \equiv \begin{cases} 
 p_S \frac{\partial \pi(p_S, c_S)}{\partial c_S} - 1 & : \pi(p_S, c_S) \geq \pi(p_I, c_I) \\
(p_I - p_S) \frac{\partial \pi(p_S, c_S)}{\partial c_S} - 1 & : \pi(p_S, c_S) < \pi(p_I, c_I)
\end{cases} = 0
\] (6)

From our assumptions it follows that there is a unique solution \((p_I^*, c_I^*)\) to equations (3) and (4), as already explained. It also follows that there is at most one solution \((p_S^*, c_S^*)\) to the equations in the respective upper lines of (5) and (6), although there is no guarantee of the existence of such a solution. An analogous argument holds for the respective lower lines of (5) and (6); if a solution \((p_S^*, c_S^*)\) exists, then it is unique. Note, superscripts “\(>\)” and “\(<\)” refer to the startup’s optimal choice given that it pursues a project of higher and lower value in case of success than the incumbent’s project, respectively.

The firms’ first-order conditions with respect to \(p\) are identical for \(\pi(p_S, c_S) \leq \pi(p_I, c_I)\), while those with respect to \(c\) are identical for \(\pi(p_S, c_S) \geq \pi(p_I, c_I)\). The intuition behind this observation is that, for \(\pi(p_S, c_S) < \pi(p_I, c_I)\), the incumbent’s R&D activity reduces the startup’s revenues compared with a monopolist’s by the multiplicative factor of \((1 - p_I)\); the startup creates revenues only if the incumbent fails. This factor affects the startup’s trade-off between revenues and costs, making, due to concavity of \(p\pi(p, c)\) with respect to \(c\), lower investments more attractive. At the same time, it leaves the trade-off between a safer and a more valuable project unchanged. Thus, if a solution to the first-order conditions in the lower lines of (5) and (6) exists, we refer to it as the startup’s “low cost strategy.” When choosing it, the startup banks on having the only successful project.

In contrast, if \(\pi(p_S, c_S) \geq \pi(p_I, c_I)\), then I’s R&D reduces S’s expected payoff compared with a monopolist’s by the subtractive term of \(p_S p_I \pi(p_I, c_I)\), thus making smaller values of \(p_S\) (entailing higher payoffs in case of success) more attractive to \(S\). At the same time, the two players’ first-order conditions with respect to investment level, \(c\), are identical (see (4) and the upper line of (6)). If a solution to the first-order conditions in the upper lines of (5) and (6) exist, we call it the startup’s “high risk strategy.”

\[5\] In this term, “risk” retains its colloquial meaning of “probability of failure.” A more accurate term, though rather awkward, would be “low success probability strategy.”
Choosing this strategy, the startup aims at having a better solution than the incumbent, and does so by pursuing riskier projects. This distinction lies at the heart of our results.

Examining the startup’s locally best responses, \((p_S^\geq (p_I, c_I), c_S^\geq (p_I, c_I))\) and \((p_S^\leq (p_I, c_I), c_S^\leq (p_I, c_I))\), illuminates the incentive effects further. Due to the envelope theorem, the total derivative of 

\[
\frac{d}{dp_I} \Pi_S(p_I, c_I, p_S^\geq (p_I, c_I), c_S^\geq (p_I, c_I))
\]

with respect to \(p_I\) and \(c_I\) is identical to the respective partial derivative; the same holds for 

\[
\frac{d}{dc_I} \Pi_S(p_I, c_I, p_S^\geq (p_I, c_I), c_S^\geq (p_I, c_I)).
\]

Thus:

\[
\frac{d}{dp_I} \Pi_S(p_I, c_I, p_S^\geq (p_I, c_I), c_S^\geq (p_I, c_I)) = -p_S \frac{d}{dp_I} (p_I \pi(p_I, c_I)) \tag{7}
\]

\[
\frac{d}{dc_I} \Pi_S(p_I, c_I, p_S^\geq (p_I, c_I), c_S^\geq (p_I, c_I)) = -p_S p_I \frac{d}{dc_I} \pi(p_I, c_I) < 0 \tag{8}
\]

\[
\frac{d}{dp_I} \Pi_S(p_I, c_I, p_S^\leq (p_I, c_I), c_S^\leq (p_I, c_I)) = -p_S \pi(p_S, c_S) < 0 \tag{9}
\]

\[
\frac{d}{dc_I} \Pi_S(p_I, c_I, p_S^\leq (p_I, c_I), c_S^\leq (p_I, c_I)) = 0 \tag{10}
\]

These relations have a clear interpretation. The incumbent’s choosing a safer project diminishes the startup’s profits from the low cost strategy (see (9)), while the effect on \(S\)’s profits at \((p_S^\geq, c_S^\geq)\) is ambiguous and vanishes for \(p_I = p_I^*\) (see (7)). The intuition is that banking on having the only successful project becomes less attractive the higher the success probability of the other player’s project. Instead, if the startup aims for a superior project, it is the expected value of the incumbent’s project that matters. This value is maximized for \(p_I = p_I^*\), and infinitesimal changes in \(p_I\) around \(p_I^*\) do not affect the expected value of the incumbent’s project. Turning to the effects of changes in \(c_I\), the incumbent’s investing more makes the high risk strategy, \((p_S^\leq, c_S^\leq)\), less attractive for \(S\) (see (8)), while its profits at \((p_S^\leq, c_S^\leq)\) are unchanged (see (10)). Also this finding is intuitive: the more valuable the incumbent’s project in case of success, the harder it becomes for the startup to realize profits by having a superior project. On the other hand, if the startup realizes profits only if the incumbent fails, then it is irrelevant how valuable the incumbent’s project is in case of success. We summarize these findings in the following proposition.
PROPOSITION 1. If solutions \((p_S^\xi(p_1, c_1), c_S^\xi(p_1, c_1))\) and \((p_S^\zeta(p_1, c_1), c_S^\zeta(p_1, c_1))\) to the startup’s first-order conditions exists, then the following holds. (i) \(p_S^\xi(p_1^*, c_1^*) < p_1^*, \) and \(c_S^\xi(p_1^*, c_1^*) < c_1^*.\) (ii) With increasing investment by the incumbent, the startup’s profits at \((p_S^\xi(p_1, c_1), c_S^\xi(p_1, c_1))\) decline while those at \((p_S^\zeta(p_1, c_1), c_S^\zeta(p_1, c_1))\) are unaffected. (iii) When the incumbent increases its success probability, then the startup’s profits at \((p_S^\xi(p_1, c_1), c_S^\xi(p_1, c_1))\) decline. Those at \((p_S^\zeta(p_1, c_1), c_S^\zeta(p_1, c_1))\) are unaffected by infinitesimal variations of \(p_1\) around the solution \(p_1^*(c_1)\) to \(I\)'s first-order condition with respect to \(p_1.\)

For the subsequent analysis we introduce the function \(\bar{c}(p),\) defined as follows:

\[
\bar{c}(p) = \begin{cases} 
0 & : \pi(p, c) > \pi(\bar{p}, \bar{c}) \ \forall c \\
\infty & : \pi(p, c) < \pi(\bar{p}, \bar{c}) \ \forall c \\
\text{solution to } \pi(p, c) = \pi(\bar{p}, \bar{c}) & : \text{otherwise}
\end{cases}
\]  

(11)

Since \(\pi(p, c)\) is, by definition, strictly increasing in \(c,\) the equation in the last line of the above expression can, for any given \(p,\) be fulfilled by at most one value of \(c.\) Inserting \(c = \bar{c}(p)\) into the lower line in (11) and differentiating implicitly with respect to \(p\) shows that the slope of \(\bar{c}(p)\) is equal to the marginal rate of technical substitution:

\[
\frac{\partial \bar{c}(p)}{\partial p} = \text{MRTS} = -\frac{\frac{\partial \pi(p, c)}{\partial p}}{\frac{\partial \pi(p, c)}{\partial c}} > 0 .
\]  

(12)

The inequality follows from the assumptions we made regarding the value function. The function \(\bar{c}(p)\) divides the \((p, c)\) space into two areas. For convenience, we introduce the following notation:

\[
A^\geq := \{ (p, c)|\pi(p, c) \geq \pi(\bar{p}, \bar{c})\} \\
A^\leq := \{ (p, c)|\pi(p, c) \leq \pi(\bar{p}, \bar{c})\}
\]  

(13)

We will also use \(A^\geq\) and \(A^\leq,\) defined as \(A^\geq\) and \(A^\leq\) but with strict inequalities.
3.2. Separable Value Functions

Before we derive additional general results, we first study the class of separable value functions, which enables further characterization of the candidate equilibria. Afterward, we look at a specific, separable value function for which the overall equilibrium of the game can be determined.

**Proposition 2.** If \( \pi(p, c) \) is separable—i.e., \( \pi(p, c) = \pi_p(p) \cdot \pi_c(c) \)—then if the incumbent plays its dominant strategy the startup’s profit function has a local maximum in the interior of \( A^* \) where \( p^*_S = p^*_I \) and \( c^*_S < c^*_I \). If a local maximum in \( A^* \) exists, it is characterized by \( p^*_S < p^*_I \) and \( c^*_S < c^*_I \).

Proposition 2 shows that the two local maxima remain separate, \( p^*_S < p^*_I \), and pursuing one or the other type of locally optimal project is a truly dichotomous choice.

We consider now the specific, separable value function, \( \pi(c, p) = (1 - p)c^\alpha \) in order to obtain additional insight. For the incumbent and a startup playing a low investment strategy, the first-order condition with respect to \( p \) simplifies to:

\[
\frac{\partial \Pi_j}{\partial p_j} = 0 \iff p^*_I = p^*_S = \frac{1}{2}.
\] (14)

The corresponding investment decisions are given by the first-order conditions:

\[
\frac{\partial \Pi_j}{\partial c_I} = c^*_I(p_I) = (\alpha p_I(1 - p_I))^{\frac{1}{1 - \alpha}}
\] (15)

\[
\frac{\partial \Pi_S}{\partial c_S} = 0 \iff c^*_S(p_S) = \left(\frac{\alpha}{2} p_S(1 - p_S)\right)^{\frac{1}{1 - \alpha}}
\] (16)

If the startup instead follows the high risk strategy, the following two first-order conditions characterize the R&D choices:

\[
\frac{\partial \Pi_S}{\partial c_S} = 0 \iff c^*_S(p_S) = (\alpha p_S(1 - p_S))^{\frac{1}{1 - \alpha}}
\] (17)

\[
\frac{\partial \Pi_S}{\partial p_S} = 0 \iff p^*_S(c_S) = \frac{1}{8c^\alpha} \left(4c^\alpha - \left(\frac{\alpha}{4}\right)^{\frac{\alpha}{1 - \alpha}}\right)
\] (18)
Figure 1 illustrates the two candidate equilibria for $\alpha = 1/2$. The incumbent has a dominant R&D strategy and chooses $p_i^* = 1/2$ and $c_i^*(1/2) = 1/64$. If the startup follows the low investment strategy, it also chooses $p_S^* = 1/2$, but invests less than the incumbent, $c_S^*(1/2) = 1/256$. The high risk strategy is located north-west of the dotted line $\tilde{c}(p)$. The startup chooses the same investment as the incumbent for a given value of $p$. However, as the startup takes a higher risk than the incumbent, it also chooses a lower investment, $p_S^* = 0.3652$ and $c_S^*(1/2) = 0.01344$. For $\alpha = 1/2$, the low cost strategy $(p_S^*, c_S^*)$ is globally optimal for the startup.

We now turn to the question of how the equilibrium changes with $\alpha$. The marginal rate of technical substitution equals

$$MRTS(\alpha) \equiv -\frac{\partial \pi(p_i, c_i)}{\partial c_i} \left/ \frac{\partial \pi(p_i, c_i)}{\partial p_i} \right. = \frac{\alpha (1 - p_i)}{c_i}. \quad (19)$$

For given $(p, c)$, the marginal rate of technical substitution is increasing in $\alpha$. In other words, an increase in $\alpha$ implies that investment becomes relatively more important for the creation of value in case of success. Therefore, the startup’s investment if it pursues the high risk strategy and aims at a superior project, increases relative to the one made if it plays the low investment strategy. In other words, $c_S^*/c_S^*$ is increasing in $\alpha$. At the same time, an increase in $\alpha$ triggers a decrease in $p_S^*$ in order to reduce the investment requirement of the high risk strategy; that is, $p_S^*/p_S^*$ is decreasing in $\alpha$. These effects imply that the high risk strategy becomes more costly for the startup compared with the low investment strategy as $\alpha$ increases. Numerically solving the model and calculating the startup’s profits for both local maxima show that two strategies yield identical payoffs for $\alpha \approx 0.3128$. For values of $\alpha$ above this threshold, the low cost strategy $(p_S^*, c_S^*)$ is globally optimal for the startup, while for smaller values the high risk
solution \((p^*_S, c^*_S)\) yields the higher payoff. The startup’s payoff at the high risk strategy becomes negative at \(\alpha \approx 0.5656\) and at \(\alpha \approx 0.7145\) a local maximum in \(A^>\) ceases to exist.

### 3.3. Existence of Two Local Maxima of the Startup’s Profit Function

In this subsection, we provide two general results regarding the startup’s profit function and its two types of strategies. While we cannot spell out or interpret in full generality the conditions under which two local maxima of the startup’s profit function exist,\(^6\) we prove two related results. The first of these refers to the slope of the startup’s profit function at the incumbent’s equilibrium strategy point.

**Proposition 3.** The gradient of \(\Pi_S(p^*_i, c^*_i, p_S, c_S)\), considered as a function of \((p_S, c_S)\), at \((p^*_i, c^*_i)\) equals \((-p^*_i \pi(p^*_i, c^*_i), 0)\) in \(A^>\), and \((0, -(p^*_i)^2 \partial \pi(p^*_i, c^*_i)/\partial c_i)\) in \(A^<\).

Proposition 3 describes a local version of the duality between a high risk and a low cost equilibrium for the startup: relative to the payoff that \(S\) realizes by mimicking the incumbent’s strategy, it can improve locally both by going for higher risk and by going for lower cost.\(^7,8\) With Proposition 3 it might seem intuitive that there should always be two distinct maxima of the startup’s profit function in \(A^>\) and \(A^<\), respectively. This need not be the case, however: it may happen that following the line of steepest ascent originating at \((p^*_i, c^*_i)\) along one of the gradients does not lead to a maximum in the respective area, but to the border between the two areas—and from there to the maximum in the other area, which then constitutes the only maximum of the startup’s profit function.

---

\(^6\) These conditions relate both to local (near \((p^*_i, c^*_i)\)) and global characteristics of the value function, \(\pi(p, c)\). The latter are highly technical and not insightful.

\(^7\) Note that it is possible for the function \(\Pi_S\) to have two different gradients at \((p^*_i, c^*_i)\) since it is not continuously differentiable along the curve \(\bar{c}(p)\), which passes through \((p^*_i, c^*_i)\).

\(^8\) A statement about which direction yields the larger increase in profits cannot be made since the two dimensions, \(p\) and \(c\), are not comparable in their scaling.
The second general result relates to an extended game, in which in an additional Stage 0 Nature decides if the incumbent does R&D or not. Let $p_{R&D}$ denote the probability that the incumbent does R&D. The startup knows this probability, but not the outcome of Nature’s move. For this situation, we prove:

**Proposition 4.** In the extended game characterized by the incumbent’s R&D probability $p_{R&D}$, the startup’s profit function has two local maxima, in $A^>$ and $A^<$, respectively, if $p_{R&D}$ is sufficiently small.

Also, Proposition 4 constitutes, in a sense, a local version of the duality between high risk and low cost equilibria. It furthermore describes a situation that will be realistic in many cases—namely, that the startup has only probabilistic assumptions about the incumbent’s performing or not performing competing R&D. In circumstances where the startup attaches a low probability to the incumbent working on a competing R&D project, the high risk and the low cost strategy are both locally optimal for the startup.

4. Extensions

We now turn to extensions of our model. We introduce bargaining power on the side of the incumbent, and we consider the case of two startups that are competing for acquisition. These extensions serve to demonstrate that our key finding of a dichotomous choice of R&D strategy by startups is robust.

4.1. Bargaining Power

In the basic model, we made the simplifying assumption that all bargaining power rests with the startup. We relax this assumption now, analyzing the general case that the parties split the surplus that the startup’s project creates over the value of the incumbent’s. Specifically, we assume that the incumbent receives the share $\beta$, $0 \leq \beta \leq 1$, of this surplus. To keep the analysis simple, we employ the specific value function used before, $\pi(p, c) = (1 - p) \sqrt{c}$. The players’ profit functions are as follows.

$$
\Pi_I(p_l, c_I, p_S, c_S) = p_I(1 - p_I) \sqrt{c_I} + \beta (1 - p_I) p_S(1 - p_S) \sqrt{c_S - c_I}
$$

$$
\Pi_S(p_l, c_I, p_S, c_S) = (1 - \beta)(1 - p_I) p_S(1 - p_S) \sqrt{c_S - c_S}
$$

\begin{equation}
\pi(p_s, c_s) < \pi(p_I, c_I)
\end{equation}

(20)
Consider first the candidate equilibrium where the incumbent pursues the project of highest value. For the first-order conditions derived from (20), it is easy to show that the incumbent no longer has a dominant strategy: its optimal strategy depends on the strategy of the startup. Solving for the candidate equilibrium, we find:

\[
\begin{align*}
\Pi_t(p_t, c_t, p_S, c_S) &= p_t (1 - p_t) \sqrt{c_t} + \beta p_S \left((1 - p_S) \sqrt{c_S} - p_t (1 - p_t) \sqrt{c_t}\right) - c_t \quad \text{;} \quad \pi(p_S, c_S) \geq \pi(p_t, c_t) \\
\Pi_S(p_t, c_t, p_S, c_S) &= (1 - \beta) p_S \left((1 - p_S) \sqrt{c_S} - p_t (1 - p_t) \sqrt{c_t}\right) - c_S
\end{align*}
\]

(21)

Inserting these solutions into \( \pi(p, c) \) shows that they fulfill the condition \( \pi(p_S^\leq, c_S^\leq) < \pi(p_t^\leq, c_t^\leq) \).

Looking first at the startup’s choices, the startup chooses \( c_S^\leq = \frac{1}{2} \) independently of \( \beta \). The startup’s investment level \( c_S^\leq \) exhibits a strong, negative dependence on \( \beta \), decreasing from \( \frac{1}{256} \) at \( \beta = 0 \) to zero at \( \beta = 1 \). This is intuitive: through the incumbent’s bargaining power the startup’s revenues decrease by a factor of \( 1 - \beta \).

Turning to the incumbent, notice that project failure is less costly compared with the base model, because the incumbent captures a fraction \( \beta \) of the value created by the startup’s project. The higher the value that the incumbent expects to capture from the startup, the more risk is the incumbent willing to take in its own project. Numerical calculations show the incumbent chooses a project with slightly lower success probability than the startup: with \( \beta \) varying from zero to unity, \( p_t^\leq \) decreases from 0.5 to \( \left(1 + \sqrt{14}/4\right)/4 \approx 0.4839 \) (at \( \beta = 0.5 \)), and then increases again to 0.5. An increase in \( \beta \) allows the incumbent to capture more value but discourages investment, and thus value creation, by the startup. As a result of these two opposing effects, the incumbent has the highest expected profit in case of project failure, and chooses the highest risk, for \( \beta = 0.5 \). The incumbent’s investment \( c_t^\leq \) also shows a u-shaped dependence on \( \beta \) as the return on R&D investment decreases the further \( p_t^\leq \) is away from \( \bar{p} = 0.5 \). It can be verified...
that these strategies constitute an equilibrium since neither the incumbent nor the startup has an incentive to deviate.

Interestingly, as one can show, \( \Pi_i (p_i^c, c_i^c, p_s^c, c_s^c) \) reaches its maximum at \( \beta = 0.5 \) where the optimal balance between value capture by the incumbent and value creation by the startup is achieved from the point of view of the incumbent. Not surprisingly, the profit of the startup is decreasing in \( \beta \).

Consider now the candidate equilibrium where the startup has the highest value in case of success. Here, the incumbent wishes to acquire a successful startup, and the acquisition price is decreasing both in the value of the incumbent’s own project and in the incumbent’s bargaining power, \( \beta \). In that sense, project value and bargaining power are substitutes from the point of view of the incumbent. Except for \( p_i^c = 0.5 \), no closed-form solutions to the first-order conditions derived from (21) exist. One can show that a candidate equilibrium satisfying \( \pi(p_s^c, c_s^c) > \pi(p_i^c, c_i^c) \) exists for \( \beta \leq 0.2173 \). However, this is not an equilibrium of the game, because it results in negative profits for the startup.

The result that only the low investment equilibrium exists is not due to the generalized assumptions regarding bargaining power, but to setting the exponent of \( c \) in the definition of \( \pi(p, c) \) equal to 0.5. For sufficiently low values of the exponent \( \alpha \) (see Section 3.2) and sufficiently small \( \beta \), the startup’s high risk strategy will exist and be globally optimal for reasons of continuity.

Thus, our finding of two local equilibria, corresponding respectively to a high risk and a low cost strategy of the startup, is robust to changes in the distribution of bargaining power, as long as the startup receives a sufficiently large share of the pie. We summarize:

**RESULT 1.** If \( \pi(p, c) = (1 - p)\sqrt{c} \) and the incumbent receives the share \( \beta \), \( 0 \leq \beta \leq 1 \), of the surplus that the startup’s project creates over the value of the incumbent’s, then: (i) For all values of \( \beta \), an equilibrium exists in which the startup pursues a low cost strategy. (ii) For \( 0 \leq \beta \leq \beta_{\text{max}} \), where \( \beta_{\text{max}} \approx 0.2173 \), a solution to the first-order conditions exists in which the startup pursues a high risk strategy. This solution does not constitute an equilibrium of the game.
4.2. Two Startups

We now study the case of one incumbent and two startups competing for acquisition. We focus on the above family of value functions, \( \pi(p, c) = (1 - p)c^a \). In the acquisition stage, the entrants simultaneously make price offers to the incumbent. If the incumbent has the most valuable project, no acquisition occurs. If this is not the case, it follows from standard Bertrand competition arguments that the incumbent acquires the most valuable project in equilibrium, paying the difference in value between the most valuable and the second most valuable project.

We will use the following notation: the two startups are denoted 1 and 2 where start-up \( i \) chooses \( c_i \) and \( p_i \). Incumbent \( I \) chooses \( c_I \) and \( p_I \). The profit of startup \( i \) can be written as:

\[
P_i (p, c) = \begin{cases} 
  p_i \left( \pi(p_i, c_i) - p_j \pi(p_j, c_j) - (1 - p_j)p_i \pi(p_i, c_i) \right) - c_i : V_i > V_j \geq V_l \\
  \left(1 - p_j\right) \left(p_i \pi(p_i, c_i) - p_j \pi(p_j, c_j)\right) - c_i : V_i \geq V_l \geq V_j \\
  \left(1 - p_i\right) \left(p_i \pi(p_i, c_i) - p_j \pi(p_j, c_j)\right) - c_i : V_i > V_l \geq V_j \\
  \left(1 - p_i\right) \left(1 - p_j\right) \pi(p_i, c_i) - c_i : \text{otherwise}
\end{cases}
\]

where \( p \) and \( c \) are vectors of investments and success probabilities. Without loss of insight, it will be assumed in the following that \( V_1 > V_2 \) in equilibrium. Then, the profit of the incumbent is given by:

\[
P_I (p, c) = \begin{cases} 
  p_1 p_2 \pi(p_2, c_2) + (1 - p_1) p_2 \pi(p_1, c_1) - c_l : V_2 \geq V_l \\
  p_1 p_2 (1 - p_l) \pi(p_2, c_2) + p_2 \pi(p_1, c_1) - c_l : \text{otherwise}
\end{cases}
\]

In contrast to the baseline model with only one startup, the incumbent’s choice \( (c_I, p_I) \) depends on the startups’ choices. Consider the case of \( V_2 > V_l \). The first-order conditions of the incumbent are:

\[
\frac{\partial P_I}{\partial p_I} \equiv \pi(p_I, c_I) + p_i \frac{\partial \pi(p_I, c_I)}{\partial p_i} = 0
\]
\[
\frac{\partial \Pi_i}{\partial c_i} \equiv (1 - p_1 p_2) p_i \frac{\partial \pi(p_i, c_i)}{\partial c_i} - 1 = 0
\]  

(27)

The first-order condition with respect to \( p_i \) is identical to that obtained in the case of only one startup. Nonetheless, the solutions will, in general, not be the same since the first-order conditions with respect to \( c_i \) differ (and \( c_i \) appears in (26)). A general statement cannot be made if the solution \( p_i^* \) to (26) and (27) is equal to \( \tilde{p} \), larger, or smaller. In contrast, the first-order condition with respect to \( c_i \) (27) does differ from the equivalent equation in the baseline model, in which the term \( (1 - p_1 p_2) \) is absent. If \( p_i^* \) is not too different from the solution in the base case then this implies a lower investment level, \( c_i^* \), than in the baseline model (since \( \pi(p_i, c_i) \) is concave in \( c \)). Intuitively, since the incumbent can acquire Startup 2 for free if both startups are successful, the fallback option of its own project becomes less important for the incumbent, resulting in less investment in R&D.

Consider now the case of \( V_i > V_2 \). Then, the first-order conditions of the incumbent are:

\[
\frac{\partial \Pi_i}{\partial p_i} = \pi(p_i, c_i) + p_i \frac{\partial \pi(p_i, c_i)}{\partial p_i} - p_1 p_2 \pi(p_2, c_2) = 0
\]

(28)

\[
\frac{\partial \Pi_i}{\partial c_i} = p_i \frac{\partial \pi(p_i, c_i)}{\partial c_i} - 1 = 0
\]

(29)

Here, the first-order condition for \( c_i \) (29) is identical to that in the baseline model, while the first-order conditions with respect to \( p_i \) differ. In analogy to the case \( V_2 \geq V_i \), a general statement regarding if and how \( c_i^* \) differs from the equilibrium value in the baseline model is not possible. If \( c_i^* \) is not too different from the solution in the baseline model, then this implies an optimal choice of \( p_i^* < \tilde{p} \). The possibility of acquiring Startup 2 at zero cost with a positive probability implies here that the incumbent takes more risk compared with a situation with only one startup, since the incumbent’s cost of a failing project is reduced.

Summing up the above discussion, competition among the startups implies that Startup 2 can be acquired at a price of zero with positive probability \( p_1 p_2 \). Depending on how valuable Startup 2’s project
is compared with the incumbent’s own project, this possibility induces either the incumbent to invest less, or to take more risk, compared with our baseline model with no competition in the market for acquisitions.

In order to better understand the difference between the incumbent’s and the startups’ choices, it is useful to make the following comparison: take the ranking of the VICOS as given and fix the R&D decisions of the other firms in the market. Then, compare the optimal R&D decisions of an incumbent and an entrant. For example, compare the decisions in a situation where the firm in question has the second largest VICOS in equilibrium. Denote the decision of the firm with the highest and lowest VICOS by \((p_H, c_H)\) and \((p_L, c_L)\), respectively. Then, the decisions of the incumbent are characterized by the first-order conditions:

\[
\frac{\partial \Pi_i}{\partial p_i} \equiv \pi(p_i, c_i) + p_i \frac{\partial \pi(p_i, c_i)}{\partial p_i} - p_H p_L \pi(p_L, c_L) = 0
\]

\[
\frac{\partial \Pi_i}{\partial c_i} \equiv p_i \frac{\partial \pi(p_i, c_i)}{\partial c_i} - 1 = 0
\]

Using equation (15), the decisions of startup \(i\) are characterized by the first-order conditions:

\[
\frac{\partial \Pi_i}{\partial p_i} \equiv \pi(p_i, c_i) + p_i \frac{\partial \pi(p_i, c_i)}{\partial p_i} - p_L \pi(p_L, c_L) = 0
\]

\[
\frac{\partial \Pi_i}{\partial c_i} \equiv (1 - p_H)p_i \frac{\partial \pi(p_i, c_i)}{\partial c_i} - 1 = 0
\]

Comparing the two pairs of first-order conditions, it follows immediately that the incumbent invests more and takes less risk than a startup. The intuition is that the incumbent benefits from its innovation not only if it is the best, but also if it is the second-best. In the latter case, the incumbent’s innovation is not implemented in equilibrium, but serves as a bargaining chip that reduces the price of acquiring the best innovation. In contrast, a startup only benefits from its innovation if it is the best in the market. Put differently, the incumbent has a stronger incentive to invest in R&D and to “play it safe” than a startup.
The same type of analysis is possible for the two remaining cases where the firm in question has the highest or the lowest VICOS. In both cases, the incumbent has a (weakly) stronger incentive to invest in R&D than a startup and a (weakly) stronger incentive to choose a high success probability.

The equilibrium of the game is characterized by a system of six non-linear first-order conditions. Furthermore, the solutions to the first-order conditions correspond to locally optimal choices for the firms. To find the Nash equilibrium, it needs to be verified that a candidate solution also represents globally optimal choices for the firms. Together, these issues imply that the model is solvable only numerically.

**RESULT 2.** There exist three types of equilibria in the model. (i) For $\alpha \in (0,0.20662]$, there exists an equilibrium for which $V_1 > V_2 > V_3$. (ii) For $\alpha \in [0.17783,0.36566]$, there exists an equilibrium for which $V_1 > V_2 > V_3$. (iii) For $\alpha \in [0.35491,0.5]$, there exists an equilibrium for which $V_1 > V_2 > V_3$.

--- Figure 2 about here ---

As argued in section 4.2, an increase in $\alpha$ favors investment over risk taking in the creation of value for the specification of $\pi(p,c)$ used. We find again that the incumbent pursues the project of highest value for high values of $\alpha$ and the project of lowest value for low values of $\alpha$. With two startups, an intermediate configuration is the equilibrium outcome for intermediate values of $\alpha$. Here, one startup plays the high risk strategy whereas the other plays the low cost strategy. Overall, the extension to two firms confirms the result that startups have the choice between two types of strategies, a low cost or a high risk strategy.

### 5. Discussion

Technology acquisitions are highly diverse with respect to their value and the radicalness of their technology. Acquisition targets may provide radically new and superior technologies, as described by
Henkel et al. (2015) for the EDA and the biotechnology industry. But they may also constitute low cost alternatives to an internal development by the incumbent, as for example is arguably the case if an incumbent internationalizes by acquiring a copycat startup in another country.

We have developed and analyzed a model of R&D competition between an incumbent and one or two startups that provides an economic explanation of this diversity. We find that the marginal incentives for the startup differ structurally between two situations. If the startup aims at being better than the incumbent, it has an incentive to pursue riskier projects than the potential acquirer. In contrast, if it banks on having the only successful project then it is optimal for the startup to invest less than the incumbent. Globally, either the high risk or the low cost strategy may be optimal for the startup. This finding is robust toward variations in the parties’ relative bargaining power, and to the introduction of a second startup. As a supplementary result, we find that having too much bargaining power may be harmful to the incumbent. Anticipating that it will receive only a small share of the surplus, the startup has limited incentives to invest in the first place, which, in turn, negatively affects the incumbent’s profits in case of an acquisition.

Our results contribute to a better understanding of the interplay of incumbents and startups in technological innovation. In particular, the established wisdom that startups tend to be better at radical innovation requires qualification. While under certain conditions startups do have an incentive to pursue R&D approaches with a higher value in case of success, there are also circumstances under which they will go after less valuable projects. The economic logic behind both types of equilibria differs from explanations provided by earlier authors. Arrow (1962) argues that an entrant has a greater incentive than an incumbent to invest in radical innovation, since the latter would cannibalize its existing product. While this situation resembles our high risk equilibrium insofar as the entrant creates the more valuable innovation, it is not higher investments that drive the outcome in our model, but rather a riskier R&D approach. This approach, we note, will often be more realistic, given that startups tend to be financially constrained. Gilbert and Newbery (1982) argue for the case of incremental innovation that an incumbent’s incentive to remain a monopolist, and thus to invest in its innovation, is greater than an entrant’s incentive to become one of two duopolists. Again, there is a superficial similarity to low cost equilibria in our
model, but the explanations differ; our result is based not on the anticipation of market duopoly, given
that the incumbent has a product with certainty, but on the likelihood of the incumbent failing.

Our study has a number of limitations. First, we exclude product market entry by the startup, such
that our model is applicable only to industries such as ICT and biotechnology, in which being acquired is
the predominant path for startups. Second, we model a bilateral monopoly and a situation where two
startups compete for acquisition, but we leave out the reverse case of two or more incumbents competing
to acquire a smaller number of startups. Third, we assume as the only difference between a startup and an
incumbent the former’s lack of complementary assets. Including cannibalization on the side of the
incumbent would make the model richer, but also difficult to handle and less focused. Finally, we provide
a model but no empirical evidence of a dichotomy between high risk and low cost startup strategies. Thus,
future work should seek empirical support for our model predictions, and should analyze how industry
characteristics translate into model parameters that favor one or the other type of equilibrium.
References


Figures

**Figure 1:** Strategy space for $\pi(p, c) = (1 - p)\sqrt{c}$ with first-order condition curves

![Strategy space diagram](image_url)

- **northwest of $c(p)$:** S’s VICOS greater than I’s
- **southeast of $c(p)$:** S’s VICOS less than I’s
- **local max of S’s profits:** $(p^*_S, c^*_S)$
- **max of I’s profits:** $(p^*_I, c^*_I) = (\bar{p}, \bar{c})$
- **global max of S’s profits:** $(p^*_S, c^*_S)$

**Figure 2:** The equilibrium with two startups and one incumbent for different values of $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0</th>
<th>0.17783</th>
<th>0.20662</th>
<th>0.35491</th>
<th>0.36566</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I: VICOS$_1 >$ VICOS$_2 >$ VICOS$_3$

II: VICOS$_1 >$ VICOS$_3 >$ VICOS$_2$

III: VICOS$_3 >$ VICOS$_1 >$ VICOS$_2$
Appendix

Proof of Proposition 2
The gradients can be directly calculated from the partial derivatives of $\Pi_S$ (equations (5) and (6)), taking into account that they are evaluated at $(p^*_i, c^*_i)$ and that this point is defined by equations (3) and (4).
Furthermore, the gradient parallel to $(-1,0)$ does indeed point into the area $A^>$ since the curve $\tilde{c}(p)$ that confines the area has a positive slope and runs through $(p^*_i, c^*_i)$. By the same argument, the gradient parallel to $(0,-1)$ points into the area $A^<$.

Proof of Proposition 3
With exogenous probability $p_{R&D}$ that the incumbent does R&D, the startup receives with probability $(1 - p_{R&D})$ a payoff described by the incumbent’s profit function. For the sake of notational clarity, we define the profit function $\tilde{\Pi}_I(p_i, c_i) \equiv p_i \pi(p_i, c_i) - c_i$. It is identical to $\Pi_I(p_i, c_i, p_S, c_S)$ as defined by (1), but makes explicit in the notation that it does not depend on $(p_S, c_S)$. With this notation, the firm’s payoff functions are:

$$
\Pi_I^{ex}(p_i, c_i) = p_{R&D} \tilde{\Pi}_I(p_i, c_i) \ ,
$$

$$
\Pi_S^{ex}(p_i, c_i, p_S, c_S) = p_{R&D} \Pi_S(p_i, c_i, p_S, c_S) + (1 - p_{R&D}) \tilde{\Pi}_I(p_S, c_S) \ .
$$

We note that the incumbent’s dominant strategy is unchanged as long as $p_{R&D} > 0$, $(p^*_i, c^*_i)$. For given $\epsilon > 0$ we define the circle of radius $\epsilon$ around $(p^*_i, c^*_i)$:

$$
S^1_{\epsilon} = \left\{(p, c) \bigg| (p - p_i^*)^2 + \left(1 - \frac{\epsilon}{c^*_i}\right)^2 = \epsilon^2 \right\} \ .
$$

We define the closed disk $\bar{D}^2_{\epsilon}$ of radius $\epsilon$ around $(p^*_i, c^*_i)$ in an analogous way. Let $\mu(\epsilon)$ be defined as

$$
\mu(\epsilon) = \max\{\Pi_S(p^*_i, c^*_i, p_S, c_S) - \Pi_S(p_i^*, c_i^*, p^*_i, c^*_i) | (p_S, c_S) \in S^2_{\epsilon} \} \ .
$$
Due to Proposition 2, with sufficiently small $\epsilon$ one can approximate $\Pi_S$ on $\overline{D}_\epsilon^2$ to $O(\epsilon^2)$ by a piecewise linear function in $(p_S, c_S)$, defined separately for $A^>$ and $A^<$. Again due to Proposition 2, $\mu(\epsilon) > 0$. Furthermore, $\mu(\epsilon)$ is also the maximum of $\Pi_S$ on the closed disk, $\overline{D}_\epsilon^2$.

We now turn to the second summand in (A2), proportional to $\Pi_I$. Let

$$v(\epsilon) = \max\{\Pi_I(p_S, c_S) - \Pi_I(p^*_i, c^*_i) | (p_S, c_S) \in S^1_\epsilon\}.$$  \hfill (A5)

Since $\Pi_I$ has its maximum at $(p^*_i, c^*_i)$ and since it is concave by assumption, $v(\epsilon) < 0$. Now choose $\pi_{R\&D}$ small enough such that

$$\pi_{R\&D} \mu(\epsilon) + (1 - \pi_{R\&D}) v(\epsilon) < 0.$$  \hfill (A6)

With this condition, $\Pi^\text{ex}_S(p^*_i, c^*_i, p_S, c_S)$ is less than $\Pi^\text{ex}_S(p^*_i, c^*_i, p^*_i, c^*_i)$ for all points $(p_S, c_S)$ on $S^1_\epsilon$, that is, the startup’s payoff on $S^1_\epsilon$ is less than its payoff at $(p^*_i, c^*_i)$. At the same time, due to Proposition 2 $\Pi^\text{ex}_S$ has one local maximum each in $A^>$ and $A^<$, located up to $O(\epsilon^2)$ in the direction of $(-1,0)$ and $(0,-1)$, respectively, from $(p^*_i, c^*_i)$. This proves the proposition.

**Proof of Proposition 4**

Consider first the low investment equilibrium. If $\pi(p, c)$ is separable then both firms’ first-order conditions in $A^<$ with respect to $p$ (equations (3) and (5), bottom) simplify to $\pi_p(p) + p \frac{\partial \pi_p(p)}{\partial p} = 0$. The solution to this equation is $p^<_S = p^*_i = \bar{p}$, independent of $c$. The corresponding first-order condition for $c_S$, in the lower line of equation (6), has a solution $c^<_S < c^*_i$ since $p\pi(p, c)$ is concave in $c$ by assumption, $(1 - p^*_i) < 1$ and, thus, the solution $c^<_S$ to the lower line of equation (6) is smaller than the solution $c^*_i$ to equation (4). Hence, at $(p^*_i, c^*_i) \in A^<$ the startup’s profit function has a local maximum where $c^<_S < c^*_i$ and $p^<_S = p^*_i$.  

27
Consider now the high risk equilibrium. The upper line of Equation (5) simplifies to \( \pi_p(p) + \int_{\text{startups}} \). Since the right hand side is positive, this implies that \( p^* > p^* \) due to concavity of \( p\pi_p(p) \). Since \( p^* \) maximizes \( p\pi_p(p) \), it follows that \( p^* \pi_p(p^*) > p^* \pi_p(p^*) \). The incumbent’s and the startup’s investment are implicitly given by \( \frac{\partial \pi_c(c)}{\partial c} = \frac{1}{p^* \pi_p(p^*)} \) and \( \frac{\partial \pi_c(c^*)}{\partial c} = \frac{1}{p^* \pi_p(p^*)} \), respectively.

Then, concavity of \( \pi_c(c) \) implies that \( c^* < c^* \).