Integration and Competition for Innovation in Science-based Industries

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Abstract

We develop a model for how innovation competition affects the organisation of research activity and property-rights allocation in science-based industries. We consider a vertical production process with division of labor between research and commercialisation. We analyse firms' incentive for integration in the presence of upstream innovation competition. Integration adversely affects integrated firms' R&D investment and creates positive externality for independent firms. For a sufficiently strong externality, a semi-integrated structure appears in equilibrium. Thus, the model can explain the coexistence of integrated and independent research firms and conforms to evidence of R&D competition in science-based industries. A non-integrated arrangement can sometimes appear in equilibrium although a semi-integrated arrangement has higher innovation probability and aggregate industry payoff, because parties that gain from integration cannot commit to compensate losing parties at the contracting stage. We analyse the effects of resource constraints and inter-customer licensing on industry structure and their implications for innovation competition.
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Abstract

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1 Introduction

Competition for innovation is essential to the growth of science-based industries.\(^1\) The starting point of our study is the observation that the organization of research activity and the allocation of ownership rights affect incentives in a competition for innovation. The division of labor between research and commercialization creates a natural vertical structure in the production process of a science-based industry. We observe a complex picture of how firms draw their boundaries along this vertical structure. Consider, for example, the biotech industry. At one end of the spectrum, there are large pharmaceutical companies that maintain in-house laboratories, pursue scientific research to discover and identify drug candidates, conduct clinical trials in multiple phases to develop and commercialize their innovations further, and finally, compete in the product market. At the other end, there are small entrepreneurial biotech firms that specialize in preclinical R&D, operate only in the early stages of a drug discovery, and are funded through venture capital and private equity. Following discoveries, these biotech firms interact with other specialized firms in a market for technology for further development and commercialization of their innovations. In between the two ends of this spectrum, multiple types of R&D contracts exist in the form of collaborations, alliances, and partnerships among research laboratories, universities, biotech firms, and pharmaceutical companies (Arora et al. 2004; Gans et al. 2008).\(^2\) These contracts pave the way for the successful transfer of knowledge and rights from parties that undertake R&D to those that commercialize. The transfer of technology through licensing occurs at various stages of a drug development process. While licensing deals mostly occur in the discovery and early development stages, the frequency of late-stage licensing deals has increased over the years (Cartwright 2013; Grabowski and Kyle 2014).\(^3\)

The organization of innovation research and allocation of property rights across upstream and downstream firms have implications for industries. These features are part of the institutional arrangements under which all participants of a typical science-based industry operate. While these arrangements affect a firm’s incentives to innovate in a competition for innovation, they are also influenced by how firms compete to innovate. The close interaction between innovation competition and institutional arrangements leads us to

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\(^1\)Pisano (2010) describes a “science-based” business as one that “attempts not only to use existing science but also to advance scientific knowledge and capture the value of the knowledge it creates.” We adhere to the same definition in this study and focus on industries in which the final product is produced through a process that involves research for creating new knowledge and subsequent investment for commercializing the value of the knowledge. Examples include the biotechnology, nanotechnology, chemical, and semi-conductor industries.

\(^2\)The pharmaceutical industry has made extensive use of the market for technology in the last 2 decades (Grabowski and Kyle 2014).

\(^3\)Reduced financial constraints give biotech firms incentives for late-stage contracting (Grabowski and Kyle 2014). Gans et al. (2008) find evidence of frictions in the market for technology in the form of high uncertainty associated with a patent’s scope and challenges in transferring tacit knowledge. Such frictions can discourage pharmaceutical firms from early-stage licensing.
the following research questions.

First, how does competition for innovation affect the organization of innovation research and property-rights allocation at an industry level? Second, how do these institutional arrangements in turn influence competition for innovation? Finally, what sorts of institutional arrangements are evident in a competitive market equilibrium, and how does the equilibrium arrangement fare in terms of economic efficiency? The existing literature acknowledges the effects of various institutional arrangements, such as property-rights allocation and the possibility of technology transfer, on innovation incentives (see, e.g., Aghion and Tirole 1994; Arora et al. 2004). Our point of departure is that we study the effects of competition for innovation on firms’ incentives to innovate when the organization of innovation research, firm boundaries, and property-rights allocations are endogenous.

To answer these research questions, we develop a simple model of competition for innovation. We consider a vertical production process with a division of labor between innovation research and commercialization. Innovation is an intermediate good. Our basic model considers a two-tier duopoly setting. We refer to firms that specialize in commercialization as customers and firms that specialize in innovation research as research units. The innovation-generating process unfolds in three stages. First, customers decide whether to integrate with research units by owning property rights of their research output. Second, research units - either integrated or independent - make non-verifiable investments in research and compete for innovation. Third, if an independent research unit succeeds, customers bargain with the research unit over the commercialization rights. In contrast, if an integrated research unit succeeds, we allow inter-customer bargaining over the commercialization rights.

Three key features of the model closely capture the innovation-generating process in a typical science-based industry. The first is related to the concept of integration. In line with the property-rights approach in the organization literature (see, e.g., Grossman and Hart 1988; Hart and Moore 1988; Aghion and Tirole 1994), we analyze integration as the allocation of ownership rights of the research output before firms engage in research. Specifically, an integrated research unit transfers the property rights of a forthcoming innovation to the corresponding integrated customer before it makes any non-verifiable investment in research. Consequently, integration occurs before an innovation is realized. By contrast, an independent research unit transfers rights after it succeeds in developing an innovation but before the commercialization process.

The concept of integration has certain implications for our analysis. For example, we deal with two types of market—one at the pre-innovation stage and the other at the post-innovation stage. The contrast between

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4Our notion of integration here is close to the concept of backward integration in the vertical integration literature (Lafontaine and Slade 2007). In backward integration, the manufacturers decide whether to “make or buy” the input.
these two markets is similar to the difference between a “market for innovation” involving transaction of intellectual property for the creation of new technology and a “market for technology” involving transactions for the use and diffusion of existing technology (Arora et al. 2001).\(^5\) Furthermore, a market for innovation in our framework includes all transactions that involve transfer of ownership rights of forthcoming innovations from the innovator to the commercializing entity. Therefore, integration in our framework refers to a wide range of situations, including in-house laboratories of pharmaceutical companies, acquisition of research firms by pharmaceutical companies, and R&D contracts between a research firm and a commercializing firm in which the research firm transfers the rights of its potential research outputs before they are realized. Thus, an early-stage licensing deal in which a biotech firm grants licenses after identifying a molecule with potential application—but before conducting research to further study and develop the drug candidate—is an example of an integrated arrangement.

The second key feature deals with how to model uncertainty in the innovation-generating process. We consider two types of uncertainty. First, we assume that making a successful innovation is a stochastic event. We present this uncertainty through a probabilistic relationship between the research investment and the time of delivery of an innovation. In addition, we assume the absence of information regarding the exact nature of innovation at the pre-innovation stage. An innovation is not well defined until it is realized. Therefore, parties cannot contract for delivery of a specific innovation. We consider an incomplete-contract framework, in which a contract describes only the allocation of property rights of a forthcoming innovation against a possible transfer fee from the licensee to the licensor.

The third feature is related to the form of competition. We model competition in the form of an innovation contest. The underlying assumption is that there is a temporary monopoly rent for the first innovator—the winner of the innovation contest.\(^6\) The assumption of the monopoly rent for the innovator is not new in the innovation literature and makes a contest an ideal framework for modeling competition. The contest in our model is productive in the sense that a participant’s effort increases both the expected prize value and its winning probability.\(^7\)

\(^5\) The United States Department of Justice in its Antitrust Guidelines for the Licensing of Intellectual Property (U.S. Department of Justice 2017) makes a similar distinction between a “market for technology” and a “market for R&D” based on the differences between (a) transaction of assets comprising “intellectual property that is licensed . . . and its close substitutes” and (b) transaction of assets comprising “R&D related to the identification of a commercializable product, or directed to particular new or improved goods or processes, and the close substitutes for that research and development.”

\(^6\) We do not explicitly model the product-market competition, as our focus remains on the competition for developing an innovation as an intermediate good. The assumption of a monopoly rent in the product market makes our analysis simple and tractable. However, equivalent results can be obtained in a model of innovation for a cost-reducing technology that can foster a producer’s competitiveness in a product-market competition with multiple producers.

\(^7\) See Scotchmer (2004) for a discussion on the roles of patent and protection in fostering innovation. See Konrad (2007) for a comprehensive analysis of contest frameworks. The contest framework brings our model close to the models studied in the literature on innovation tournaments (Schmidt 2008). In these models, a firm often has a double incentive for investing in R&D – to increase
We characterize the competitive equilibrium of the innovation game and show that two types of R&D arrangements can arise. In one, which we refer to as no integration, all research units compete for innovation while retaining the property rights of their research output. The successful firm licenses its technology to customers in the post-innovation stage. In the other, which we refer to as semi-integration, one of the two research units (some but not all research units in a framework with more than two firms) sells the ownership rights of its research output to a potential customer before it competes for innovation with the other research unit that retains the ownership rights of its output. The possibility of semi-integration arrangement in equilibrium is driven by the fact that integration creates a positive externality to other firms operating in the market. Further, we evaluate various arrangements in terms of economic efficiency and show that no integration can sometimes occur in equilibrium even though a semi-integration arrangement can generate higher aggregate payoffs.

A semi-integration arrangement in equilibrium is interesting for two reasons. First, starting from a symmetric environment, we find that in equilibrium the property rights are allocated in both the upstream and downstream sectors of the market. Second, an integrated customer in the downstream market can acquire innovation from two sources: an independent research unit that retains the property rights over successful innovation (an external source) and the customer’s internal research unit that succeeds in innovation race (an internal source). Evidence from science-based industries supports both these features. For instance, in 2002, the top 10 largest pharmaceutical companies conducted the majority of their development projects in-house while buying 47 per cent of their development candidates from external sources, such as biotech firms and universities (Pisano 2006). Biotech start-ups and university lab spin-offs often compete intensively for novel science-based technologies that can subsequently be commercialized by pharmaceutical corporations (Stern 1995). Technological competition between research-focused firms and firms undertaking both research and commercialization is common in other innovative industries (Gans and Stern 2003; Norbäck and Persson 2009).

1.1 Related Literature

Our study relates to several strands of literature. This research contributes to the literature on the effects of integration and competition on innovation incentives (Aghion and Tirole 1994; Brocas 2003; Chen and Sappington 2010; Liu 2016). Aghion and Tirole (1994) explain how the allocation of property rights can affect R&D investment in industries with innovative product market. In their framework, investments in the chance of winning and the reward from winning.

\footnote{Examining 4,057 pharmaceutical projects by the 40 largest pharmaceutical companies, Guedj (2005) shows that the novelty of drugs from in-house R&D was not statistically different from drugs obtained from other sources.}
both upstream and downstream markets are necessary for innovation. While we share the role of property rights in providing incentive, our focus is on the competition for innovation as an intermediate good and how the contest-like competition affects investment in the upstream market. Chen and Sappington (2010) consider the role of the product-market competition on the incentives for innovation in the upstream market. However, we consider the product-market competition in a reduced form. In a similar framework, Brocas (2003) studies incentives to integrate and its implications for R&D investments. Unlike our model, upstream firms can simultaneously innovate and use substitutable technologies and the author focuses on the effect of switching costs on the R&D investment. In Liu (2016), integration has a positive coordination effect that boosts payoffs of the integrated units. The decision to integrate critically depends on the relative relevance of investment in the upstream and downstream markets. On the other hand, we consider a direct negative effect of integration on the integrated firms. Our work complements these studies by analyzing the role of integration in innovation incentives while sharing a common vertical production process. However, we differ in the way we model the competition for innovation and explain integration based on its positive externality on non-integrated firms.

The innovation literature has focused on the question of coexistence of research-focused firms and firms specializing in both research and commercialization in the context of science-based industries. Their inter-relationship has often been examined from the perspective of either collaboration between small and large firms (Acs and Audretsch 1988; Baumol 2010) or competition between entrant and incumbent firms (Gans and Stern 2000; Norbäck and Persson 2009). Our study explains the coexistence in an otherwise symmetric framework of competition. We recognize a new role for large corporations in competition for innovation. These corporations generate a positive externality for independent research-focused firms’ R&D efforts. Because of the positive externality, semi-integration can generate a larger surplus than no integration. Our results support the coexistence of startups and large corporations but from a different perspective to the previous literature.

Our model shares common features with models in the literature on vertical integration and foreclosure, which began with the seminal works of Salinger (1988) and Ordover et al. (1990). The primary concern in this literature has been the strategic use of foreclosure to alter the market power and price- and quantity-setting abilities of firms in competition. These models typically associate integration with potential supply constraints for independent downstream customers (Salinger 1988; Bolton and Whinston 1993; Chen 2001) or demand constraints for independent upstream suppliers (Stefanadis 1997). Thus, integration provides the integrated customer and supplier with rent-seeking opportunities. We differ from these models in one
critical aspect—we model that integration does not impose any restriction on the non-integrated firms. We allow both inter-customer trading and trading between an integrated customer and an independent innovator. A firm’s bargaining power in a transaction at the post-innovation market for technology solely depends on the distribution of the customers’ values of an innovation, which is independent of the industry structure. Thus, strategic foreclosure is not a reason for integration in our model.

We also contribute to the contest literature (Konrad 2007). Contest models often consider specific success functions, such as the Tullock success function, owing to their axiomatic foundations (Tullock 1980; Skaperdas 1996). The contest success function in our model has a game-theoretic foundation. It is derived from an underlying game in which innovation is an uncertain event and players strategically exert effort. The conditional contest-success probability given that an innovation is realized takes an additive form and coincides with the Tullock success function for a suitable choice of innovation probability. To our advantage, the contest success function is multiplicatively separable in efforts. This makes the derivation of marginal effects of effort on contest-success probabilities and payoffs easy and tractable. The framework is particularly useful in modeling contests in which the efforts are productive and the value of the contest prize is uncertain.

2 The model

We consider a game with four players—two upstream research units, $RU_1$ and $RU_2$, and two downstream customers, $C_1$ and $C_2$.\(^\text{10}\) The research units perform research necessary to realize an innovation. The customers only can commercialize an innovation. The game proceeds in three stages: pre-innovation contracting of ownership rights, an innovation contest, and post-innovation bargaining.

2.1 Pre-innovation contracting in a market for innovation

We consider an incomplete contract framework, similar to that considered in Aghion and Tirole (1994).\(^\text{11}\) The exact nature of innovation is unknown at the contracting stage and thus, the value of an innovation is not contractible. A research unit’s effort is also not contractible. A contract can specify only the allocation of ownership rights of any forthcoming innovation.

In stage 1, $C_1$ and $C_2$ simultaneously offer prices $p_1$ and $p_2$, respectively. The two research units observe prices and decide whether or not to sell ownership rights of any forthcoming innovation. If a research unit

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\(^{10}\)We have discussed the results in a model with more than two players in both markets in Section 3.

\(^{11}\)In comparison to Aghion and Tirole (1994), in our model, the bargaining power lies with the customers. In this sense, the model is more aligned with the notion of backward integration than forward integration.
sells the right, it receives no further reward when an actual innovation is realized. We call this a case of integration, and refer to the corresponding customer-research unit pair as integrated. We assume that a customer (or a research unit) can be integrated only with one research unit (or one customer). If a customer or research unit is not integrated, we refer to it as independent.

As customers offer prices simultaneously, we must specify a matching mechanism by which a research unit is matched with a customer in an integration arrangement. Consider a price profile \((p_1, p_2)\). First, suppose that both research units are willing to integrate at the highest offered price. We then randomly select one research unit and match it with the customer offering the highest price. If the other research unit is willing to sell the rights at the second highest price, it is matched with the other customer. Next, suppose that no research unit is willing to integrate at the highest offered price. Then, there is no integration. Finally, suppose that only one research unit is willing to integrate at the highest offered price. We then match the willing research unit with the customer offering the highest price. The other research unit remains independent. If two customers offer the same price, we randomly select one customer as the one offering the highest price, and follow the abovementioned matching procedure.\(^{12}\)

### 2.2 Innovation contest

Next, consider the stage in which research units engage in research competition. We normalize the minimum effort to zero. The probability of a research unit developing an innovation for a given effort level \(e \in [0, 1]\) is given by an increasing function \(q(e) \in [0, 1]\). The cost of effort is given by an increasing function \(c(e)\) with \(c(0) = 0\). Additional assumptions are needed to support the first-order approach in various scenarios. We defer discussions of those to Section 3.2.

We model research competition in the form of an innovation contest. Consider time in an interval \([0, 1]\). At the beginning of time, both research units simultaneously incur effort cost. Effort cost is sunk and cannot be altered once the contest begins. Effort is non-verifiable and therefore, non-contractible in our model. We interpret effort as applications of researchers’ knowledge and skills that are not easily measurable or verifiable and can be driven only by output-related incentive. Assuming that effort is invested only at the beginning of the contest makes the analysis simple and tractable. A research unit wins the contest if it comes up with an innovation ahead of its competitor in the fixed time interval \([0, 1]\). Let \(x_i\) denote the time

\(^{12}\)We assume that the customer offering the second highest price does not renegotiate its price offer after one of the two research units is integrated with the customer offering the highest price. We make this assumption to keep our analysis simple. The assumption, however, does not affect our results in any significant way. This is because in our model, when the customer offering the highest price becomes integrated, the other customer does not gain any additional advantage in dealing with the independent research unit, as an independent research unit will always have the option to sell its innovation to an integrated customer in the post-innovation stage.
that \( RU_i \) takes to develop an innovation. We assume that \( x_i \) follows a uniform distribution over the time interval \([0, \frac{1}{q(e_i)}]\), so that the probability that \( RU_i \) develops an innovation within the time interval \([0, 1]\) is exactly \( q(e_i) \). If no research unit innovates within the time interval \([0, 1]\), the contest ends at time 1 with no innovation. Otherwise, the contest ends at the time when a research unit comes up with an innovation ahead of its competitor.

For a given effort profile \( e = (e_1, e_2) \) such that \( e_i \) denotes \( RU_i \)'s effort level, \( RU_i \)'s probability of winning the innovation contest is
\[
\pi_i(e) = \Pr \left[ x_i = \min \{x_1, x_2\} \leq 1 \right] = \frac{1}{q(e_1)} \int_0^1 q(e_1) \left(1 - t q(e_2)\right) dt = q(e_1) \left(1 - \frac{q(e_2)}{2}\right),
\]
(1)

and
\[
\pi_2(e) = q(e_2) \left(1 - \frac{q(e_1)}{2}\right).
\]
(2)

Note that \( \pi_1(e_1, e_2) = \pi_2(e_2, e_1) \). The sum of these winning probabilities is the probability of realizing an innovation. We denote the innovation probability for a given effort profile \( e \) by \( \pi_{\text{inv}}(e) \). Therefore,
\[
\pi_{\text{inv}}(e) = 1 - \left(1 - q(e_1)\right) \left(1 - q(e_2)\right).
\]

### 2.3 Post-innovation bargaining in a market for technology

The game moves to the post-innovation bargaining stage when an innovation contest ends with a successful innovation. If an independent research unit wins the contest, it has the ownership right of the innovation and can bargain with customers over a licensing fee. If an integrated research unit wins the contest, the corresponding integrated customer has the ownership right of the innovation and can either commercialize the innovation or bargain with the other customer over a licensing fee. Let \( v_i \) denote \( C_i \)'s value of a successful innovation. We assume that \( v_i \) follows a symmetric distribution around its expected value \( \bar{v} \), and \( v_1 \) and \( v_2 \) are independently distributed. Let \( v_{\text{max}} = \max \{v_1, v_2\} \) and \( v_{\text{min}} = \min \{v_1, v_2\} \).

We model the bargaining game in reduced form. For simplicity, we consider symmetric Nash-bargaining payoff. Specifically, when a seller (either an independent research unit or an integrated customer) with a reservation value of the innovation \( r_s \) trades with a buyer (a customer) with an innovation value \( r_b \), we assume the additional value \( (r_b - r_s) \) is equally split between the seller and buyer. Therefore, the payoffs of the buyer and seller are \( \frac{r_b - r_s}{2} \) and \( \frac{r_b + r_s}{2} \), respectively.\(^{13}\) When an integrated customer sells the commercialization

\(^{13}\)The assumption of equal split of the additional rent is common in the innovation literature (Aghion and Tirole 1994).
right to the other customer (such a possibility might arise if the corresponding integrated research unit wins
the innovation contest), the integrated customer’s reservation value is its own valuation of the innovation.
On the other hand, when an independent research unit sells the commercialization right to one customer, its
reservation value is the innovation value realized by the other customer.

2.4 Payoffs and solution concept

We assume that all players are risk neutral.

The ex post payoff of $RU_i$ is given by

$$U_{RU}^{RU} = \begin{cases} p - c(e_i) & \text{if } RU_i \text{ is integrated} \\ \frac{v_{\max} + v_{\min}}{2} - c(e_i) & \text{if } RU_i \text{ is independent and wins the contest} \\ -c(e_i) & \text{if } RU_i \text{ is independent and does not win the contest} \end{cases}$$

where $p$ is the price at which $RU_i$ sells the ownership right of any forthcoming innovation and $e_i$ is the effort
level of $RU_i$.

The ex post payoff of $C_i$ is

$$U_{C}^{C} = \begin{cases} \frac{v_i - v_{\min}}{2} & \text{if } C_i \text{ is not integrated} \\ \frac{v_{\max} + v_i}{2} - p & \text{if } C_i \text{ is integrated with some } RU_j \text{ that wins the contest} \\ \frac{v_i - v_{\min}}{2} - p & \text{if } C_i \text{ is integrated with some } RU_j \text{ that does not win the contest} \end{cases}$$

where $p$ is the price at which $C_i$ buys the ownership right of any forthcoming innovation and $v_i$ is $C_i$’s
realized innovation value. We consider the subgame perfect Nash equilibrium in pure strategies as the
solution concept. We focus only on pure strategies owing to their analytical tractability.

We make some simplifying assumptions for analytical tractability. For instance, we consider fixed
transfer fees in the exchange of property rights in the market for innovation. We do not consider non-
linear contracts, which are not uncommon in practice. For example, research units might receive a bonus if
an innovation is commercialized. Such an incentive can positively affect R&D effort. However, as long as
the bonus level differs from the ex post bargaining payoff of an independent research unit, there will be a
difference in the effort levels chosen by two types of firms. We are interested in studying the impact of this
difference, which can be analyzed with more tractability in our simple setting.

In addition, we assume that an integrated customer has little control over its integrated research unit’s
choice of effort in the innovation contest. There are several reasons for this assumption. First, we interpret effort as applications of researchers’ knowledge and skills. It might not be perfectly measurable given the uncertain nature of the innovation process. Furthermore, we assume zero effort is associated with a positive success probability to reflect that an integrated customer can still control the routinized activities. Finally, our concept of integration is broad and covers various R&D arrangements. Thus, integration includes contracted research whereby a research firm can operate on its own even after selling the rights of its research output. In this case, it is expected that the research unit decides its own R&D investment.

3 Equilibrium analysis

3.1 Post-innovation bargaining

We first introduce two notations that will be useful in our subsequent analysis. As $v_i$ s are independent and symmetrically distributed around the expected value $\bar{v}$, it can be shown that $E\left(\frac{v_{\text{max}}+v_{\text{min}}}{2}\right) = \bar{v}$. We denote $E\left(\frac{v_{\text{max}}-v_{\text{min}}}{2}\right)$ by $\gamma$. Together, we can write

$$E(v_{\text{max}}) = \bar{v} + \gamma, \quad E(v_{\text{min}}) = \bar{v} - \gamma.$$

$\bar{v}$ and $\gamma$ are important parameters in our model. While $\bar{v}$ measures an innovation’s expected value, $\gamma$ measures the bargaining power of customers in the bargaining game. A customer’s bargaining power is low when valuations have high positive correlation. In this case, $v_{\text{max}}$ is likely to be close to $v_{\text{min}}$, or equivalently, $\gamma$ is close to zero.

The innovation contest can lead to three different cases: (i) an independent research unit wins the innovation contest, (ii) an integrated research unit wins the innovation contest, and (iii) the innovation contest results in no successful innovation.

First, consider that an independent research unit wins the contest. The winning research unit bargains with the customer that has the maximum valuation and its reservation value is the second highest valuation. The winning research unit’s expected payoff is given by $E\left(\frac{v_{\text{max}}+v_{\text{min}}}{2}\right) = \bar{v}$. The losing research unit has zero

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14 The distribution function and density function of $v_{\text{max}}$ are given by $G(x) = F^2(x)$ and $g(x) = 2f(x)F(x)$, where $F$ and $f$ denote the distribution function and density function of $v_i$, respectively. In addition, the distribution function and density function of $v_{\text{min}}$ are given by $H(x) = 1 - (1 - F(x))^2$ and $h(x) = 2f(x)(1 - F(x))$, respectively. Therefore,

$$E\left(\frac{v_{\text{max}}+v_{\text{min}}}{2}\right) = \int \frac{xf(x)(F(x)+1-F(x))}{2} dx = \int xf(x) dx = \bar{v}.$$

15 Precisely, $\gamma = E\left(\frac{v_{\text{max}}-v_{\text{min}}}{2}\right) = \int xf(x)(2F(x)-1) dx$. 

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payoff at this stage. A customer has an expected payoff of \( \frac{1}{2}E \left( \frac{v_{\text{max}} - v_{\text{min}}}{2} \right) = \frac{v}{2} \).

Second, consider that an integrated research unit wins the contest. The corresponding integrated customer that owns the innovation can either commercialize it or bargain with the other customer if the other customer has higher valuation. Therefore, the expected payoff of the winning customer is \( E \left( \frac{v_{\text{max}} + v_{\text{win}}}{2} \right) = \frac{v}{2} + \frac{v}{2} \). The expected payoff of the independent customer is \( E \left( \frac{v_{\text{indep}} - v_{\text{min}}}{2} \right) = \frac{v}{2} \). Each of the two research units has zero expected payoff. The aggregate expected payoff of all four players is \( v_{\text{max}} \). Inter-customer trading of licenses rules out the possibility of an ex post inefficient situation in which an innovation is not commercialized at the maximum possible value.

Finally, if an innovation contest leads to no successful innovation, the bargaining game is trivially resolved with each player having zero expected payoff at the post-innovation stage.

### 3.2 Innovation contest

We now solve for the optimal effort levels in the contest. Our first assumption below is sufficient (although not necessary) to ensure that we can find a unique solution in various scenarios by solving the first-order condition. Formally, we assume:

**Assumption 1.** \( vq(e) - c(e) \) is strictly concave in \( e \).

Assumption 1 is typically satisfied if the cost function \( c(e) \) is sufficiently convex compared to the success function \( q(e) \). Our second assumption ensures that the solution lies in the open interval \((0,1)\). Formally, we assume:

**Assumption 2.** \( vq'(1) - c'(1) > 0 < \frac{v}{2}q'(0) - c'(0) \).

In the remainder of our paper, Assumptions 1 and 2 hold unless explicitly stated.\(^{16}\)

We can have three different industry structures at the beginning of an innovation contest: (i) both research units are integrated, (ii) one of the research units is integrated while the other is not, and (iii) no research unit is integrated. We call these three structures full integration (FI), semi-integration (SI), and no integration (NI), respectively.

First, consider the case of full integration. As the integrated research units get zero payoff at the post-innovation stage, they exert no effort. By (1) and (2), the winning probabilities of the two research units \( q(0) \left( 1 - \frac{q(0)}{2} \right) \) are identical.

\(^{16}\)Assumptions 1 and 2 do not affect our results in any significant way. If we relax Assumption 1, we have to deal with multiple solutions and subsequently, with an equilibrium selection problem. If we relax Assumption 2, we have a boundary solution, which makes the solution insensitive to changes in the parameter values to some extent.
Next, consider the case of semi-integration. At this stage, without loss of generality, we assume that RU$_2$ is integrated with C$_2$, and RU$_1$ remains independent. Therefore, RU$_2$ exerts no effort, as it receives zero payoff at the post-innovation stage. RU$_1$’s expected payoff is $\overline{v} q (e_1) \left(1 - \frac{q_0}{2}\right) - c (e_1)$. The optimal effort of RU$_1$, denoted by $e^{SI}$, satisfies the following first-order condition:

$$\overline{v} \left(1 - \frac{q_0}{2}\right) q' (e^{SI}) - c' (e^{SI}) = 0.$$  \hfill (3)

Finally, consider the case of no integration. In a symmetric Nash equilibrium, the optimal effort levels of both research units, denoted by $e^{NI}$, solve the following condition:

$$e^{NI} = \arg\max_{e \in [0,1]} \overline{v} q(e) \left(1 - \frac{q(e^{NI})}{2}\right) - c (e).$$

From the first-order condition, $e^{NI}$ satisfies

$$\overline{v} \left(1 - \frac{q(e^{NI})}{2}\right) q'(e^{NI}) - c'(e^{NI}) = 0.$$ \hfill (4)

By differentiating (3) and (4), and applying Assumption 1, we find that $e^{SI}$ and $e^{NI}$ are increasing in $\overline{v}$. Furthermore, a comparison of the effort levels shows that $e^{SI} > e^{NI} > 0$. Integration influences research units’ incentives to exert effort differently. It dampens the integrated research unit’s effort as the property rights theory suggest. However, it confers a positive externality on the independent research unit: the unit exerts more effort in semi-integration than in no integration. The following lemma formally proves this observation.

**Lemma 1.** $e^{SI} > e^{NI} > 0$.

*Proof.* First, note that $e^{NI} > 0$ by Assumption 2. Denote $\left(1 - \frac{q_0}{2}\right)$ and $\left(1 - \frac{q(e^{NI})}{2}\right)$ by $A$ and $B$, respectively. We obtain $A > B$ as $e^{NI} > 0$. Note that $e^{NI}$ solves $\overline{v} B q'(e^{NI}) - c'(e^{NI}) = 0$. As $A > B$ and $q'(e^{NI}) > 0$, we must obtain $\overline{v} A q'(e^{NI}) - c'(e^{NI}) > 0$. Furthermore, note that $e^{SI}$ solves $\overline{v} A q'(e^{SI}) - c'(e^{SI}) = 0$. By Assumption 1, $\overline{v} A q(e) - c(e)$ is strictly concave and therefore, we must obtain $e^{NI} < e^{SI}$. \qed

Below, we provide an example with a specific form of linear success function $q(e)$.

**Example 1.** Consider the following distribution of customer valuation: $v_i$ takes two values, 3 or 1, each with 0.5 probability. Then, $\overline{v} = 2$ and $\overline{v} = 0.5$. Let $c(e) = e^2$ and $q(e) = \frac{1-\alpha}{4} + \frac{3+\alpha}{4} e$ for $\alpha \in [0,1]$. In Figure 1, we consider $\alpha = 0.8$. The two straight lines plot the best-response functions of the two research units in
Lemma provides an important insight into the effect of contest on innovative effort. In semi-integration, an

\[ \pi_{\text{SI}}(e_1, e_2) \]

denote the innovation probabilities, computed at the optimal effort profile, in the cases of full integration, semi-integration, and no integration by \( \pi_{\text{FI}}(e_1, e_2) \). The dotted curves present the choices of \( e_1 \) and \( e_2 \) at which the innovation probability \( \pi_{\text{inv}}(e_1, e_2) \) is constant. For \( \alpha = 0.8 \), we obtain \( \pi_{\text{FI}}(e^{NI}, e^{NI}) = 0.882 \) and \( \pi_{\text{SI}}(e^{SI}, 0) = 0.933 \). Figure 2 plots the response functions for \( \alpha = 0.4 \). For \( \alpha = 0.4 \), we obtain \( e^{NI} = 0.578 \), \( e^{SI} = 0.786 \), \( \pi_{\text{FI}}(e^{NI}, e^{NI}) = 0.871 \), and \( \pi_{\text{SI}}(e^{SI}, 0) = 0.846 \).

As illustrated in Example 1, the effect of integration on the innovation probability can be ambiguous. We denote the innovation probabilities, computed at the optimal effort profile, in the cases of full integration, semi-integration, and no integration by \( \pi_{\text{FI}}^{SI}, \pi_{\text{SI}}^{SI}, \) and \( \pi_{\text{NI}}^{SI}, \) respectively. We obtain

\[
\begin{align*}
\pi_{\text{FI}}^{SI} &= \pi_{\text{inv}}(0, 0) = 1 - (1 - q(0))^2, \\
\pi_{\text{SI}}^{SI} &= \pi_{\text{inv}}(e^{SI}, 0) = 1 - (1 - q(0)) \left(1 - q(e^{SI})\right), \\
\pi_{\text{NI}}^{SI} &= \pi_{\text{inv}}(e^{NI}, e^{NI}) = 1 - (1 - q(e^{NI}))^2.
\end{align*}
\]

The innovation probability is the lowest in the full integration case. However, the comparison between \( \pi_{\text{SI}}^{SI} \) and \( \pi_{\text{NI}}^{SI} \) is ambiguous. Define

\[ \Delta_{\text{inv}} := \pi_{\text{SI}}^{SI} - \pi_{\text{NI}}^{SI}. \]

**Lemma 2.** \( \Delta_{\text{inv}} \geq 0 \) if and only if \( (1 - q(0)) \left(1 - q(e^{SI})\right) \leq (1 - q(e^{NI}))^2 \). Furthermore, \( \pi_{\text{FI}}^{SI} \leq \min\{\pi_{\text{SI}}^{SI}, \pi_{\text{NI}}^{SI}\} \).

The proof follows by comparing \( \pi_{\text{FI}}^{SI}, \pi_{\text{SI}}^{SI}, \) and \( \pi_{\text{NI}}^{SI} \) and by the fact that \( e^{SI} > e^{NI} > 0 \). The above two lemma provides an important insight into the effect of contest on innovative effort. In semi-integration, an
independent firm faces weak competition from the integrated firm. However, weak competition can sometimes boost R&D investment of the independent firm and results in higher aggregate innovation probability. In the following example, we compare the innovation probabilities for a particular class of linear success function.

**Example 2.** We continue with the same parameter specification considered in Example 1. We assume that 

\[ q(e) = \frac{1-\alpha}{4} + \frac{3+\alpha}{4} e \]  

for \( \alpha \in [0, 1] \). Figure 3 plots the innovation probabilities \( \pi_{inv}^{FI}, \pi_{inv}^{SI}, \) and \( \pi_{inv}^{NI} \) as functions of \( \alpha \) and illustrates the results of Lemma 2: the innovation probability is lower in semi-integration than in no integration for smaller values of \( \alpha \) and is higher otherwise. It also shows that the innovation probabilities are higher in both semi-integration and no integration than in full integration for all \( \alpha \).

### 3.3 Pre-innovation contracting

At the pre-innovation contracting stage, the research units decide whether to integrate after observing a price profile \((p_1, p_2)\). Three different industry structures might arise in equilibrium. Table 3.3 presents the payoffs of the research units and customers in different structures. Without loss of generality, we assume that \(RU_2\) and \(C_2\) are integrated in semi-integration. In the following subsections, we analyze equilibrium possibilities in various structures.

**Table 3.3: Payoff in different market structures**

<table>
<thead>
<tr>
<th></th>
<th>Full integration</th>
<th>No integration</th>
<th>Semi-integration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>1 (independent)</td>
</tr>
<tr>
<td>RU</td>
<td>(p)</td>
<td>(p)</td>
<td>(\forall/2) (\pi_{inv}^{NI}) - (c(e^{NI}))</td>
</tr>
<tr>
<td>(\forall/2) (\pi_{inv}^{FI}) (=) (\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
<td>(\forall/2) (\pi_{inv}^{FI}) (=) (\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
<td>(\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
<td>(\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
</tr>
<tr>
<td>(-p)</td>
<td>(-p)</td>
<td>(-p)</td>
<td>(-p)</td>
</tr>
<tr>
<td>(C)</td>
<td>(\forall/2) (\pi_{inv}^{FI}) (=) (\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
<td>(\forall/2) (\pi_{inv}^{FI}) (=) (\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
<td>(\forall/2) (\pi_{inv}^{SI}) (=) (\forall/2) (\pi_{inv}^{NI})</td>
</tr>
</tbody>
</table>

#### 3.3.1 Equilibrium with full integration

In an equilibrium with full integration, both customers must offer the same price. This is because the customer offering the higher price otherwise has a strict incentive to decrease the offered price without
affecting the chance to integrate. Denote the common price by \( p \). In full integration, a research unit has an incentive to integrate if \( p \) is higher than its opportunity cost of integration, which is given by \( v \pi_{SI}^2(0, e) - c(e) \). The customer is only willing to offer a price \( p \) below its relative benefit from integration (derived in the proof of Lemma 3), which is given by \( (v/2) \pi_{FI}^{FI} + (v/2) \pi_{FI}^{FI} - (v/2) \pi_{SI}^{SI} \). Therefore, an equilibrium with full integration exists for some price \( p \) if and only if

\[
\forall \pi_2(0, e) - c(e) \leq \frac{v}{2} \pi_{FI}^{FI} - \frac{v}{2} (\pi_{SI}^{SI} - \pi_{FI}^{FI})
\]

(6)

In this case, the optimal price \( p \) coincides with the lower bound of (6). The following lemma documents the finding.

**Lemma 3.** An equilibrium with full integration exists if and only if condition (6) holds.

A formal proof is given in the appendix. The following proposition shows that the condition required for the existence of an equilibrium with full integration is not satisfied for any parameter values.

**Proposition 1.** There is no competitive equilibrium with full integration.

**Proof.** The inequality (6) can be rewritten as

\[
\frac{v}{2} (\pi_{SI}^{SI} - \pi_{FI}^{FI}) \leq \frac{v}{2} \pi_{FI}^{FI} - (\forall \pi_2(0, e) - c(e))
\]

The left-hand side is always positive, as \( \pi_{SI}^{SI} > \pi_{FI}^{FI} \). However, the right-hand side is always negative, as \( \forall \pi_2(0, e) - c(e) > \forall \pi_2(0, 0) - c(0) = \frac{v}{2} \pi_{FI}^{FI} \). Hence, (6) cannot be satisfied.

The mechanism behind this result is as follows. The contracted price is a transfer between a research unit and a customer. Therefore, in any equilibrium, a customer-research unit pair must be able to maximize the joint payoff. In full integration, a customer-research unit pair receives a joint payoff of \( \frac{v + v}{2} \pi_{FI}^{FI} \). However, they can deviate to a semi-integration arrangement, which provides this pair a higher joint payoff given by \( \forall \pi_1(e, 0) - c(e) + \frac{v}{2} \pi_{SI}^{SI} \).

### 3.3.2 Equilibrium with no integration

In a no-integration equilibrium, neither research unit is willing to integrate at the maximum price. This happens when a customer’s benefit from integration when other customer is not integrated is less than a research unit’s payoff in no integration. The condition, which is derived in the proof of Lemma 4, is given...
by
\[
\pi_2 (e^{SI}, 0) + \frac{\nu}{2} \pi_{inv}^{SI} - \frac{\nu}{2} \pi_{inv}^{NI} \leq \frac{\nu}{2} \pi_{inv}^{NI} - c (e^{NI})
\]
\[
\frac{\nu}{2} (\pi_{inv}^{SI} - \pi_{inv}^{NI}) \leq \pi_2 (e^{NI}, e^{NI}) - c (e^{NI}) - \pi_2 (e^{SI}, 0)
\]
\[
\Delta_{inv} \leq \frac{2\eta}{\nu}, \text{ where}
\]
\[
\eta = \pi_2 (e^{NI}, e^{NI}) - c (e^{NI}) - \pi_2 (e^{SI}, 0).
\]

(7)

The parameter \( \eta \) measures the difference between an independent research unit’s payoff in a no-integration equilibrium and an integrated research unit’s payoff in a semi-integration equilibrium. It is easy to observe that \( \eta \) is always positive and increasing in \( \nu \).\(^{17}\) The condition (7) is violated only if \( \pi_{inv}^{SI} \) is sufficiently greater than \( \pi_{inv}^{NI} \). The following lemma shows that condition (7) is a necessary and sufficient condition to have an equilibrium with no integration. A formal proof is given in the appendix.

**Lemma 4.** An equilibrium with no integration exists if and only if condition (7) holds.

### 3.3.3 Equilibrium with semi-integration

Without loss of generality, assume that in a semi-integration equilibrium arrangement, \( RU_2 \) is integrated with \( C_2 \), and \( RU_1 \) and \( C_1 \) are not integrated. Then, \( p_2 \) must lie in between \( RU_2 \)’s opportunity cost of integration, which is \( \frac{\nu}{2} \pi_{inv}^{NI} - c (e^{NI}) \) and \( C_2 \)’s benefit from integration in semi-integration, which is \( \pi_2 (e^{SI}, 0) + \frac{\nu}{2} \pi_{inv}^{SI} - \frac{\nu}{2} \pi_{inv}^{NI} \). Thus, the condition to have semi-integration in equilibrium is

\[
\pi_2 (e^{SI}, 0) + \frac{\nu}{2} \pi_{inv}^{SI} - \frac{\nu}{2} \pi_{inv}^{NI} \geq \frac{\nu}{2} \pi_{inv}^{NI} - c (e^{NI})
\]
\[
\Delta_{inv} \geq \frac{2\eta}{\nu}.
\]

(8)

The following lemma shows that (8) is a necessary and sufficient condition to have an equilibrium with semi-integration. In such an equilibrium, we have \( p_2 = \frac{\nu}{2} \pi_{inv}^{NI} - c (e^{NI}) \) and the optimal response of \( C_1 \) is to offer a price strictly below \( p_2 \). A formal proof of the lemma is given in the appendix.

**Lemma 5.** An equilibrium with semi integration exists if condition (8) holds.

The following proposition characterizes all competitive equilibria in pure strategies.

---

\(^{17}\) We obtain \( \pi_2 (e^{NI}, e^{NI}) \geq \pi_2 (e^{NI}, 0) \geq \pi_2 (e^{SI}, 0) \), where the first inequality follows from the fact that \( e^{NI} \) is \( RU_2 \)'s best response given that \( RU_1 \) exerts \( e^{NI} \) levels of effort and the second inequality follows from the fact that \( e^{SI} > e^{NI} \). Application of the envelope theorem shows that \( \pi_2 (e^{NI}, e^{NI}) - c (e^{NI}) \) is increasing in \( \nu \). As \( \pi_2 (e^{SI}, 0) \) decreases with \( \nu \), \( -\pi_2 (e^{SI}, 0) \) is increasing in \( \nu \). Together, we find that \( \eta \) is increasing in \( \nu \).
**Proposition 2.** A competitive equilibrium always exists. The equilibrium exhibits semi-integration if and only if (8) holds. Otherwise, we have no integration in equilibrium.

The proof directly follows from the preceding discussion, and therefore, is skipped.

Semi-integration is likely to occur if the innovation probability is sufficiently higher in semi-integration than in no integration. To understand the mechanism, consider a market structure with no integration initially. A research unit (assume RU₂) is willing to integrate if the price is as high as its payoff under no integration. RU₂’s decision to integrate, however, increases RU₁’s rent-seeking motivation and subsequently, RU₁’s effort level in contest. An increase in RU₁’s effort increases not only its expected payoff, but also all customers’ expected payoff, if the aggregate innovation probability is higher in semi-integration than in no integration. If the increase in innovation probability is sufficiently high, the customer C₂ can improve its payoff from integration, even after paying RU₂ its asking price.¹⁸ Semi-integration is observed in our framework, because integration creates a positive externality for the independent research unit’s effort in an innovation contest and subsequently, benefits all customers by increasing the aggregate innovation probability.

The effect of the (expected) innovation value on the existence condition (8) is not necessarily unidirectional. This is because the difference in innovation probability, Δinv, can move in both directions with a change in v. However, the effect of the parameter v is straightforward. For high values of v, semi-integration is less likely to arise. Recall that v reflects the customer’s bargaining power. For a very specialized innovation, in which only one customer can possibly commercialize with high surplus, v is high. Therefore, semi-integration is less likely to occur in such a case.

Although our basic model considers a duopoly setting in both upstream and downstream markets, the model can be extended easily to a setting with more than two firms in each market. While increasing the number of firms does not bring any new tradeoffs in our model, we find our results are robust to such an extension. In a framework with m firms in the upstream market and n firms in the downstream market, industry structure with various degrees of semi-integration is possible. In a semi-integration arrangement with k ∈ {0, 1, 2, ..., min {m, n}} integration, an independent research unit’s optimal effort eSI solves

\[
\tau q'(eSI) \left(1 - \frac{q(0)}{2}\right)^k \left(1 - \frac{q(eSI)}{2}\right)^{m-k-1} - c'(eSI) = 0.
\]

We find that in this general framework, a full integration arrangement cannot arise. The intuition is similar

¹⁸Note that the customer also receives a premium from integration, which is given by τπ₂(eSI, 0). However, the premium is never sufficient to compensate for the research unit’s opportunity cost of integration.
to that found in the duopoly setting. In full integration, no independent research unit can benefit from the positive externality of integration. The total surplus is too small to compensate for the loss that any integrating customer incurs from the low effort of its own integrated research unit. The model predicts a unique equilibrium in which there is either no integration or semi-integration where some but not all research units are integrated.

4 Discussion

4.1 Efficiency

Because of inter-customer licensing, the customer with the maximum valuation always commercializes the innovation in our model. We write the social value of an innovation at an effort profile \((e_1, e_2)\) as

\[
W(e_1, e_2) = E(v_{\text{max}}) \pi_{\text{inv}}(e_1, e_2) - c(e_1) - c(e_2).
\]

Define \(W^*\) as \(\max_{(e_1, e_2) \in [0,1]^2} W(e_1, e_2)\). As utility can be transferred among players at no cost in our model (through contracted prices and licensing fees), an outcome is efficient only if the aggregate payoff is as high as \(W^*\).\(^{19}\) For tractability, we assume that the social value of innovation is concave in efforts.

Assumption 3. \(W(e)\) is strictly concave in \(e\).

Assumption 3 is satisfied if the cost function \(c(e)\) is sufficiently convex. From the first-order condition, we can characterize the symmetric, value-maximizing effort profile \((e^w, e^w)\) such that \(e^w\) satisfies:

\[
(\bar{v} + \bar{v})(1 - q(e^w))q'(e^w) - c'(e^w) = 0.
\]

(10)

Assumption 2 implies that \(e^w > 0\), and therefore, full integration cannot be an efficient outcome. The convexity of the cost function implies that the asymmetric distribution of effort costs reduces the social value of an innovation. Therefore, semi-integration cannot be an efficient outcome. A no-integration outcome, on the other hand, has a symmetric distribution of costs. However, it can result in more or less than the socially optimal effort. The following lemma shows that a no-integration outcome can be socially wasteful if the success probability at the optimal no-integration effort is sufficiently high.

Lemma 6. \(e^w \leq e^{NI}\) if and only if \(\frac{2\bar{v}}{\bar{v} + \bar{v}} \leq q(e^{NI})\).

\(^{19}\)In other words, for any outcome with aggregate payoff less than \(W^*\), we can always construct another Pareto-dominant payoff profile yielding higher social value of innovation.
Proof. In the Appendix.

Two opposite forces drive the abovementioned result. On the one hand, rent-seeking incentives can lead to excess efforts. On the other hand, ex-post bargaining with customers reduces incentive to exert effort. It is easy to observe that $e^{NI}$ is increasing in $\bar{v}$ and does not depend on $v$. On the other hand, $\frac{2v}{\bar{v}+2\bar{v}}$ is decreasing in $\bar{v}$ and increasing in $v$. Taken together, we can conclude that in no integration equilibrium research units exert more effort than socially optimal effort level if $\bar{v}$ is sufficiently high or if $v$ is sufficiently low.

\textbf{Corollary 1.} $e^w \leq e^{NI}$ if and only if $\bar{v}$ is sufficiently high or if $v$ is sufficiently low.

Thus, the model predicts that if an innovation is highly valuable or if customers’ valuations are positively correlated, a no-integration arrangement is likely to be socially wasteful. Therefore, for large-scale innovations and for less specialized innovations (such that multiple customers can commercialize the innovation), no integration is likely to lead to socially wasteful effort.

While all three integration arrangements are potentially inefficient, we undertake a welfare comparison among them below. In particular, we study whether an equilibrium arrangement can be dominated by another arrangement in terms of social value of innovation. The social value of an innovation, computed at the optimal effort profile in the innovation contest, in the cases of full integration, semi-integration and no integration are given by

\begin{align*}
W^{FI} &:= W(0,0) = (\bar{v}+v)\pi^{FI}_{inv} \\
W^{SI} &:= W(e^{SI},0) = (\bar{v}+v)\pi^{SI}_{inv} - c(e^{SI}) \\
W^{NI} &:= W(e^{NI},e^{NI}) = (\bar{v}+v)\pi^{NI}_{inv} - 2c(e^{NI}),
\end{align*}

respectively. It is easy to see that the social value of an innovation in full integration is always dominated by the social value of an innovation in semi-integration; however, the comparison of the social value between semi-integration and no integration is ambiguous. In particular,

\begin{align*}
W^{SI} \leq W^{NI} &\iff \Delta_{inv} \leq \frac{c(e^{SI}) - 2c(e^{NI})}{\bar{v}+v}.
\end{align*}

The right-hand side expression in (12) measures the difference in total effort between semi-integration and no integration per unit of the expected maximum value of an innovation.\textsuperscript{21} As the condition in (12) differs

\textsuperscript{20}With non-productive contest, rent-seeking incentives always lead to socially wasteful effort. Chung (1996) considers a special type of productive contest, in which contest prize is a function of aggregate efforts, and shows that players exert more than the socially optimal effort in equilibrium.

\textsuperscript{21}Note that the maximum customer valuation, $v_{max}$, has an expected value of $\bar{v} + v$.  

from the equilibrium characterizing condition in (8), it is obvious that the competitive equilibrium might not necessarily maximize the industry payoff. The following proposition shows that a no-integration can appear in equilibrium while a semi-integration arrangement can yield higher aggregate payoff.

**Proposition 3.** A no-integration arrangement can appear in equilibrium while a semi-integration arrangement can yield higher aggregate payoff if the following holds:

\[
\frac{(c(e^{SI}) - 2c(e^{NI}))}{v + \gamma} \leq \Delta_{inv} \leq \frac{2\eta}{\gamma}.
\]

By contrast, if a semi-integration arrangement appears in equilibrium, it results in higher aggregate payoff than a no-integration arrangement.

**Proof.** In the Appendix.

An important insight from the efficiency analysis is that even when a no-integration arrangement is observed in equilibrium, it does not necessarily yield higher aggregate industry payoff than semi-integration does. Why do we then observe a no-integration arrangement in equilibrium? A particular arrangement can be sustained in equilibrium as long as each pair of a research unit and a customer cannot become better off by deviating to an alternate arrangement. Starting from a situation of no-integration, a pair’s decision to integrate reduces their joint payoff but creates a positive externality to the independent research unit’s effort. However, the independent research unit cannot commit to compensate the integrated customer the loss from integration at the contracting stage. Therefore, customers end up restricting themselves from offering higher prices that can lead to integration.

### 4.2 Resource constraints

Up to now, we have not assumed any potential resource constraints on the customers and research units, which could have implications for our analysis. To illustrate these implications, below we model the credit constraints in a very simple manner.

Consider first the possibility that research units face borrowing constraints. Suppose that a research unit can borrow up to \( c(L) > 0 \). If \( L \geq e^{SI} \), the constraint has no impact on the equilibrium arrangement. If \( e^{NI} \leq L < e^{SI} \), then the effort profile in no integration is unaffected; however, the effort of an independent research unit in semi-integration reduces to \( L \).\(^{22}\) Such reduction can adversely affect the possibility of semi-integration in equilibrium. If, for example, \( RU_2 \) and \( C_2 \) are integrated in semi-integration, the claimed price

\(^{22}\)An independent research unit’s payoff in semi-integration is strictly increasing in the range \([0, L]\) for any \( L < e^{SI} \).
for integration must lie above $RU_2$’s opportunity cost of integration and below $C_2$’s benefit from integration.

In this case, $RU_2$’s opportunity cost of integration is $\pi_2 (e^{NI}, e^{NI}) - c (e^{NI})$ and $C_2$’s benefit is $\pi_2 (L, 0) + \frac{1}{2} (\pi_1 (L, 0) + \pi_2 (L, 0)) - \frac{1}{2} (\pi_1 (e^{NI}, e^{NI}) + \pi_2 (e^{NI}, e^{NI}))$. After rearranging terms, we obtain the following condition to sustain semi-integration in equilibrium:

$$\pi_2 (L, 0) + \frac{1}{2} (\pi_1 (L, 0) + \pi_2 (L, 0)) \geq \frac{1}{2} \pi_{inv}^{NI} + \pi_2 (e^{NI}, e^{NI}) - c (e^{NI}).$$ (13)

At $L = e^{SI}$, condition (13) coincides with the condition of a semi-integration equilibrium (8). Furthermore, it can be shown that the expression on the left-hand side of (13) is increasing in $L$ and the condition is not satisfied at $L = e^{NI}$. Therefore, a borrowing constraint can adversely affect the possibility of semi-integration in equilibrium as $L$ decreases and when $e^{NI} \leq L < e^{SI}$.

As $L$ further decreases further (i.e., $L$ between 0 and $e^{NI}$), the effort of an independent research unit in both semi-integration and no integration reduces to $L$. The corresponding equilibrium condition for the existence of semi-integration changes to the following:

$$\pi_2 (L, 0) + \frac{1}{2} (\pi_1 (L, 0) + \pi_2 (L, 0)) \geq \frac{1}{2} \pi_{inv}^{FI} + \pi_2 (e^{NI}, e^{NI}) - c (L).$$ (14)

The abovementioned condition in (14) is not satisfied for any value of $0 \leq L < e^{NI}$. In this case, only no-integration equilibrium survives. It is noteworthy that we still do not see full integration in equilibrium for any $L > 0$. The condition for existence of full integration in equilibrium is

$$\pi_1 (L, 0) - c (L) \leq \frac{1}{2} \pi_{inv}^{FI} - \frac{1}{2} (\pi_1 (L, 0) + \pi_2 (L, 0)) - c (e^{NI}),$$ (15)

which is not satisfied for any $L > 0$. Thus, borrowing constraints for research units increase the possibility of no integration relative to semi-integration in equilibrium. This is because a borrowing constraint dampens the impact of the positive externality of integration.

A resource constraint on the customer’s side can be introduced in a similar way, such that there is an exogenous upper bound on the price that it can offer at the pre-innovation contracting stage. The resource constraint does not affect the possibility of no integration in any adverse way, as customers do not pay any price upfront in a no-integration arrangement. However, the resource constraint can adversely affect the possibility of semi-integration in equilibrium if the customer cannot arrange funds as high as the research unit’s reservation price, which is $\pi_{inv}^{NI} - c (e^{NI})$. In such a situation, we observe no integration in equilibrium.

We must interpret our findings from the above analysis with caution. Our model assumes no fixed cost of
participating in the innovation contest. If we allow such fixed cost and if research units are heavily resource constrained, then only full integration can be observed. However, in the absence of fixed cost, we find that the adverse effects of resource constraints in the upstream markets are more prominent on a semi-integrated arrangement than on a no-integration arrangement.

4.3 Inter-customer trading

Inter-customer licensing is not uncommon in practice (Arora et al. 2004). However, to find its effect in our model, we here investigate the model outcome in the absence of inter-customer trading.

The possibility of inter-customer trading has no direct impact on a research unit’s incentive to exert effort. However, this possibility can change customers’ expected payoffs at the pre-innovation contracting stage. To observe this, consider an event in which an integrated research unit (e.g., RU_2) wins the innovation contest and the corresponding integrated customer (e.g., C_2) has low valuation of the innovation. The other customer (in this case, C_1) receives zero payoff in the absence of inter-customer trading, even if the value of its innovation is high. As before, full integration cannot be an equilibrium even if inter-customer trading is not allowed. The following condition for full integration equilibrium is never satisfied for any parameter values:

\[
\left( \frac{v}{2} \right) \pi_1 (e^{SI}, 0) \leq \left( \frac{v}{2} \right) \pi_{inv}^{FI} - \left( \frac{v}{2} \pi_1 (e^{SI}, 0) - c(e^{SI}) \right).
\]  

(16)

The absence of inter-customer trading, however, reduces the possibility that semi-integration becomes an equilibrium. Comparing the research unit’s opportunity cost of integration with the customer’s relative benefit from integration, we can derive the following condition that uniquely determines whether we observe semi-integration or no integration in equilibrium:

\[
\begin{align*}
\frac{v}{2} \left( \pi_1 (e^{SI}, 0) - \pi_{inv}^{NI} \right) &\geq \left( \pi_{inv}^{FI} - \pi_1 (e^{SI}, 0) - c(e^{SI}) - \frac{v}{2} \pi_{inv}^{NI} \right) \\
\frac{v}{2} \pi_1 (e^{SI}, 0) + \frac{v}{2} \pi_2 (e^{SI}, 0) &\geq \left( \pi_{inv}^{FI} - \pi_2 (e^{SI}, 0) - c(e^{SI}) - \frac{v}{2} \pi_{inv}^{NI} \right) \\
\frac{v}{2} \pi_{inv}^{SI} - \frac{v}{2} \pi_{inv}^{NI} &\geq \eta + \frac{v}{2} \pi_2 (e^{SI}, 0) \\
\Delta_{inv} &\geq \frac{2\eta}{v} + \pi_2 (e^{SI}, 0).
\end{align*}
\]

(17)

If (17) is satisfied, we observe semi-integration in equilibrium; otherwise, no integration is observed in equilibrium. Comparing (17) with (8), we find that if inter-customer trading is ruled out, semi-integration is less likely to be sustained in equilibrium.
4.4 Innovation contest

4.4.1 Contest success function

Contest success function (CSF) is an important part of modeling a contest. Because of its strong axiomatic foundation, many contest models assume a CSF of the following additive form:

\[
p_i(e_1, \ldots, e_n) = \begin{cases} 
\frac{f(e_i)}{\sum_{j=1}^{n} f(e_j)} & \text{if } \max \{f(e_1), \ldots, f(e_n)\} > 0 \\
\frac{1}{n} & \text{otherwise,}
\end{cases}
\]  

(18)

where \( f \) is a positive, increasing function. Skaperdas (1996) shows that any CSF with the following five axiomatic properties—imperfect discrimination, monotonicity, anonymity, consistency, and independence—must be of the additive form (18).\(^{23}\)

We derive a research unit’s winning probability from an underlying environment, in which we treat innovation as a probabilistic event. Therefore, a player’s winning probability is composed of two factors—probability of developing an innovation and probability of winning the contest. In an innovation contest, an independent RU\(_i\) chooses \( e_i \) to maximize payoff, \( \pi_i(e_i, e_j) - c(e_i) \), where \( \pi_i(e_i, e_j) \) is RU\(_i\)’s winning probability. We can rewrite the payoff as follows:

\[
\bar{v} \cdot \pi_i(e_i, e_j) - c(e_i) = (\bar{v} \cdot \pi_{inv}(e_i, e_j)) \pi_i(e_i, e_j) / \pi_{inv}(e_i, e_j) - c(e_i),
\]  

(19)

where \( \bar{v} \pi_{inv}(e_i, e_j) \) is the expected value of an innovation and \( \pi_i(e_i, e_j) / \pi_{inv}(e_i, e_j) \) is RU\(_i\)’s contest-success probability given that an innovation is realized. The contest-success probability \( \pi_i(e_i, e_j) / \pi_{inv}(e_i, e_j) \) satisfies all five desired axiomatic properties, and therefore, can be expressed in an additive form (18):

\[
\frac{\pi_i(e_i, e_j)}{\pi_{inv}(e_i, e_j)} = \frac{q(e_i) \left(1 - \frac{q(e_j)}{2}\right)}{q(e_i) \left(1 - \frac{q(e_i)}{2}\right) + q(e_j) \left(1 - \frac{q(e_j)}{2}\right)}
\]

\[
= \frac{f(e_i)}{\sum_{j=1}^{n} f(e_j)},
\]

where \( f(e_i) = \frac{q(e_i)}{1 - \frac{q(e_i)}{2}} \).\(^{24}\) Thus, we generalize the additive-CSF framework in modeling contest with

---

\(^{23}\)Clark and Riis (1998) extend the result to non-anonymous CSF.

\(^{24}\)With suitable choice of \( q(e_i) \) (e.g., consider \( q(e_i) = \frac{2e^r_i}{e^r_i + 1} \)), the contest-success probability coincides with the Tullock CSF (Tullock 1980).
uncertain prize. An advantage of our model is that the winning probability \( \pi_i(e_i, e_j) = q(e_i) \left( 1 - \frac{q(e_j)}{2} \right) \)
is multiplicatively separable, which makes it easy to derive the marginal effects of a player’s effort on the winning probability and payoff.

### 4.4.2 Productive contest

An innovation contest is a productive contest. In a non-productive contest, a player’s effort contributes only to her success in winning the contest. By contrast, in a productive contest, a player’s effort increases both the expected prize value and her success probability. Therefore, a player’s incentive to exert effort differs between a productive contest and a non-productive contest. This observation has implications for our findings regarding the existence of semi-integration equilibrium.

Semi-integration occurs in equilibrium only if the innovation probability is sufficiently higher in semi-integration than in no integration. Lemma 2 shows that the condition \( e_{SI} > e_{NI} \) is a sufficient condition for higher innovation probability in semi-integration. In a two-player contest, this condition implies that a player’s effort level is higher when the other player exerts no effort than when both players exert effort. Below, we show that a productive contest is essential for the condition \( e_{SI} > e_{NI} \) to hold. Specifically, we show that a contest with fixed prize value and an additive-CSF cannot satisfy the condition \( e_{SI} > e_{NI} \).

Consider a general two-player productive contest framework. The payoff to player \( i \in \{1, 2\} \) is

\[
v(e_1, e_2) p_i(e_1, e_2) - c(e_i),
\]

where \( v(e_1, e_2) \) is the prize value and increasing in \( e_i \), \( p_i(e_1, e_2) \) is \( i \)'s contest-success probability, and \( c(e_i) \) is \( i \)'s cost of effort. We assume that \( p_i(e_1, e_2) \) can be written in additive form as \( p_i(e_1, e_2) = \frac{f(e_i)}{f(e_1) + f(e_2)} \); \( i \in \{1, 2\} \) and \( f \) is an increasing function. For simplicity, we assume a symmetric framework: two players have the same value function \( v \) and the same cost function \( c \). We further assume that the payoff is strictly concave in \( e_i \) so that we follow the first-order approach. Let \( b(e) \) denote the best response of player \( i \), given the other player’s effort \( e \). We denote the symmetric equilibrium effort by \( e^* \), and therefore, \( b(e^*) = e^* \).

A necessary condition for \( e^* \) to be less than \( b(0) \) is \( \frac{\partial v(e^*, 0)}{\partial e_1} \cdot p_1(e^*, 0) - \frac{\partial v(e^*, e^*)}{\partial e_1} \cdot p_1(e^*, e^*) > 0 \). For a non-productive contest, \( \frac{\partial v(e_1, e_2)}{\partial e_1} = 0 \) at any \( (e_1, e_2) \). Hence, a non-productive contest can never satisfy the necessary condition stated in Lemma 6. Example 3 discusses two Tullock-contest models, one with productive effort and the other with non-productive effort. The purpose of this example is to illustrate how shapes of the best-response curves differ between the two types of contest. Similar to the example

\[25\] A proof of this statement is available with the authors.
of a Tullock contest with productive effort, the innovation contest in our model generates decreasing best-response functions.

**Example 3.** Consider a two-player contest with a simple Tullock-CSF \( p_i(e_1, e_2) = \frac{e_i}{e_1 + e_2}, i = 1, 2 \). Let \( c(e) = \frac{1}{2}e^2 \). Figure 4 plots the best-response curves in the case of non-productive effort with a fixed prize value (set at 1). The best-response curves are concave and increasing at effort levels close to zero. Figure 5 plots the best-response curves when the prize value is given by \( v_i = e_i \). The response curves in this case are decreasing.

The patent race models are related to the productive-contest models. In the classic model of patent race, as pioneered by Loury (1979) and Dasgupta and Stiglitz (1980) (referred as the LDS model hereafter), multiple firms compete for a patent. Time is costly in the sense that an early discovery is better than a late discovery. An implication of the positive time-discount factor is that if a firm expands its effort (keeping others’ effort at a fixed level), the firm increases not only its chance of winning the patent, but also its payoff from winning as the expected time of discovery reduces. It can be shown that if the discount factor is sufficiently higher than zero, the LDS models can also exhibit decreasing response curves—when one firm reduces effort, the other firm responds by increasing effort.

## 5 Conclusion

Our model shows that under certain conditions integrated and independent R&D arrangements coexist in equilibrium. The theoretical implications of this finding are interesting from multiple perspectives. First,
the result explains the existence of an integrated arrangement even in a framework in which integration has a direct dampening effect on non-verifiable research investment. While this loss of incentive to exert innovative effort can indeed be a disadvantage for integration, we can explain the integration between a customer and a research firm via the presence of other research firms sharing a common environment. The positive externality of an integration arrangement on the independent firm’s investment drives the possibility of integration. Second, integration occurs in the absence of other known driving forces, including uncertainty and credit constraints in the upstream market. The model predicts integration with risk-neutral agents. Credit constraints are often cited as a possible reason behind integration (Grabowski and Kyle 2014). We find a seemingly opposite effect of resource constraints on integration. An independent research firm, if resource constrained, cannot expand its effort to the fullest extent while competing with an integrated research unit in an innovation contest. Therefore, the possibility of semi-integration in equilibrium typically reduces with the extent of resource constraints in the upstream market.

All equilibrium structures can be socially inefficient. A semi-integration arrangement leads to an asymmetric distribution of industry-wide R&D costs, which are not socially desirable with a convex cost function. The efficiency of a no-integration arrangement, however, can be ambiguous. On one hand, rent-seeking incentives in an innovation contest typically lead to over-investment of R&D effort. On the other hand, the possibility of bargaining with customers at the post-innovation stage reduces incentives for R&D investment. These two effects collectively determine the optimal effort level in a no-integration arrangement. We show that the first effect dominates the second effect if a potential innovation has a high expected value or if the customers have low bargaining power. In these cases, no integration produces socially wasteful investment. The efficiency analysis shows no integration can arise in equilibrium even when a semi-integration arrangement generates higher aggregate industry payoffs. This is because parties that benefit from an integration cannot necessarily commit to compensate those that lose in any credible way.

Our findings have important implications for policymaking. First, mergers and acquisitions are often matters of concern for their potential depressing effects on consumer welfare. In science-based industries, the question is even more complex because industry structure can affect innovation frequency and long-term growth. Our study highlights a specific externality of integration that can impact the innovation frequency in an industry. Competition policies should consider this effect when evaluating the role of integration. Second, in our model, the motivation for innovation comes from the underlying competition to grab the reward of being the first innovator. However, the capacity to innovate depends on institutional factors and constraints. In fact, resource constraints in the upstream market can severely limit the effect of positive externality generated from integration. The findings emphasize the importance of credit availability to en-
entrepreneurial research-focused firms in boosting their efforts in the competition for innovation.

We make some simplifying assumptions to keep our analysis tractable. For example, we assume a frictionless market for technology, which is not always observed in practice (Gans et al. 2008). Our current study does not address many other issues, including the complementarity of R&D investment in innovation research and the multi-stage innovation process, which are equally important for the growth of science-based industries. Although we draw many of our illustrative examples from the biotech and pharmaceutical industries, our results should be applicable to any industry in which the final products need research-based inputs, and successful innovation can give the innovator significant rent in competition. Examples of applicable industries include nanotechnology, chemical, and semi-conductor industries.

References


Appendix

We focus on pure strategies only. Let \( p = (p_1, p_2) \) denote a price profile. \( RU_i \)'s integration strategy is given by a tuple \( \text{int}_i = (\text{int}_i^f, \text{int}_i^s) \). The first component \( \text{int}_i^f(p) \) is \( RU_i \)'s integration decision at \( p_{\text{max}} \), the maximum price of the price profile \( p \). The second component \( \text{int}_i^s(p) \) is \( RU_i \)'s integration decision at \( p_{\text{min}} \), the second highest price of the price profile \( p \), given that the other research unit is integrated with the customer offering the highest price \( p_{\text{max}} \). \( \text{int}_i^f(p) \) and \( \text{int}_i^s(p) \) take binary values, 0 and 1, such that the value 1 corresponds to a decision to integrate. Let \( \text{int} = (\text{int}_1, \text{int}_2) \) denote a profile of integration strategies. \( RU_i \)'s effort strategy is to choose \( e_i \in [0, 1] \), given a price-integration strategy profile \( (p, \text{int}) \). A pure strategy of \( RU_i \) is given by \( \sigma_i = (\text{int}_i, e_i) \). Below, we present the proofs omitted from the main text.

**Proof of Lemma 3:**

**Proof.** If an equilibrium with full integration exists, both research units are willing to integrate at both prices \( p_1 \) and \( p_2 \). In such a case, both customers must offer the same price, as otherwise, the customer offering the higher price can increase her payoff by lowering the price. Denote the common price by \( p \). As \( RU_2 \) is integrated at \( p \) when \( RU_1 \) is already integrated, we must have

\[
p \geq v \pi_2 (0, e^{SI}) - c (e^{SI}). \tag{20}
\]

\( C_2 \)'s expected payoff in this equilibrium is \((v + \frac{v}{2}) \pi_2 (0, 0) + \frac{v}{2} \pi_1 (0, 0) - p \). The first component is \( C_2 \)'s expected payoff when \( RU_2 \) wins the contest times the probability that \( RU_2 \) wins the contest. The second component is \( C_2 \)'s expected payoff when \( RU_1 \) wins the contest times the probability that \( RU_1 \) wins the contest. On the other hand, if \( C_2 \) deviates by lowering its price, its expected payoff is \( \frac{v}{2} (\pi_1 (0, e^{SI}) + \pi_2 (0, e^{SI})) = \frac{v}{2} \pi^{SI}_{\text{inv}}. \) Therefore, the no-deviation condition for \( C_2 \) is given by

\[
p \leq v \pi_2 (0, 0) - \frac{v}{2} (\pi^{SI}_{\text{inv}} - \pi^{FI}_{\text{inv}}). \tag{21}
\]

From (20) and (21), we observe that a necessary condition to have an equilibrium with full integration is that

\[
v \pi_2 (0, e^{SI}) - c (e^{SI}) \leq v \pi_2 (0, 0) - \frac{v}{2} (\pi^{SI}_{\text{inv}} - \pi^{FI}_{\text{inv}}). \tag{22}
\]

The above condition is sufficient to have an equilibrium with full integration. To observe this, we construct an equilibrium as follows. Let us denote \( v \pi_2 (0, e^{SI}) - c (e^{SI}) \) by \( A \) and \( v \pi_2 (e^{NI}, e^{NI}) - c (e^{NI}) \) by
B. We have \( B \leq A \) as \( \forall \pi_2 \left( 0, e^{SI} \right) - c \left( e^{SI} \right) \geq \forall \pi_2 \left( 0, e^{NI} \right) - c \left( e^{NI} \right) \). Consider the following strategies. \( C_i \) chooses \( p_i = A \). The integration strategies \( \left( \left( int_1^f \left( p \right), int_1^s \left( p \right) \right), \left( int_2^f \left( p \right), int_2^s \left( p \right) \right) \right) \) by the research units are as follows:

\[
\begin{align*}
int_1^f \left( p \right) &= \begin{cases} 
1 & \text{if } p_{\text{max}} \geq A \\
0 & \text{otherwise}
\end{cases} \\
int_2^f \left( p \right) &= \begin{cases} 
1 & \text{if } p_{\text{max}} \geq B \\
0 & \text{otherwise}
\end{cases} \\
int_1^s \left( p \right) = int_2^s \left( p \right) &= \begin{cases} 
1 & \text{if } p_{\text{min}} \geq A \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\] (23)

In addition, \( RU_i \)'s effort strategy \( e_i \) is as follows:

\[
e_i = \begin{cases} 
0 & \text{if } RU_i \text{ is integrated} \\
e^{NI} & \text{if both research units are not integrated} \\
e^{SI} & \text{otherwise}
\end{cases}
\] (24)

**Claim 1:** The integration strategies and effort strategies given by (23) and (24), respectively, are Nash equilibrium strategies in the subgame induced by the price profile \( p \).

We have shown in section 3.2 that \( e_i \)'s are the Nash equilibrium effort strategies in the innovation contest. We now show that for a given price profile \( p \), \( \text{int}_1 = \left( int_1^f \left( p \right), int_1^s \left( p \right) \right) \) is \( RU_i \)'s optimal integration strategy given that \( RU_2 \) follows \( \text{int}_2 = \left( int_2^f \left( p \right), int_2^s \left( p \right) \right) \) and vice versa. First, note that \( RU_i \) prefers to integrate when \( RU_j, j \neq i \) if and only if \( p_{\text{min}} \geq A \). Next we claim that \( int_1^f \) is \( RU_1 \)'s best response against \( RU_2 \)'s first-stage integration strategy \( int_2^f \) and vice versa. For any price profile with \( p_{\text{max}} \geq A \), both research units’ dominant strategies are to integrate and for any profile with \( p_{\text{max}} < B \), both research units’ dominant strategies are not to integrate. Finally, if \( p_{\text{max}} \) lies in the interval \([B,A]\), and if one of the research units is integrated, the other research unit’s optimal strategy is not to integrate and vice versa. This completes the proof of Claim 1.

Furthermore, note that by offering \( p_i = A \), the customers can induce both firms to integrate, and given condition (22), none of the customers can improve the payoff by lowering its offered price when the other customer offers a price equal to \( A \). Hence, the abovementioned strategies constitute a subgame perfect Nash equilibrium.
Proof of Lemma 4:

Proof. In an equilibrium with no integration, both research units must be willing not to integrate at the maximum price. Consider a price profile \((p_1, p_2)\). When RU_1 does not integrate, RU_2’s payoff from integrating is \(\max \{ p_1, p_2 \} \) and from not integrating is \(\nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI})\). If an equilibrium with no integration exists, it must yield

\[
\max \{ p_1, p_2 \} \leq \nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI}) .
\]  

(25)

Next consider the customers’ incentive to offer low prices. Suppose \(C_1\) offers a price \(p_1 \leq \nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI})\). If \(C_2\) also offers a price \(p_2 \leq \nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI})\), its expected payoff is \(\frac{v}{2} \pi_{inv} (e^{NI}, e^{NI}) = \frac{v}{2} \pi_{inv}^{NI}\).

If \(C_2\) deviates by increasing its price above \(\nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI})\), then one of the two research units (without loss of generality, assume RU_2) chooses to accept the offer. Then, \(C_2\)’s expected payoff is given by \((\nabla + \frac{v}{2}) \pi_2 (e^{SI}, 0) + \frac{v}{2} \pi_1 (e^{SI}, 0) - p_2 = \nabla \pi_2 (e^{SI}, 0) + \frac{v}{2} \pi_{inv}^{SI} - p_2\). Thus, the no-deviation condition for \(C_2\) is given by

\[
p_2 \geq \nabla \pi_2 (e^{SI}, 0) + \frac{v}{2} \pi_{inv}^{SI} - \frac{v}{2} \pi_{inv}^{NI} .
\]  

(26)

Hence, a necessary condition to have an equilibrium with no integration is

\[
\nabla \pi_2 (e^{SI}, 0) + \frac{v}{2} \pi_{inv}^{SI} - \frac{v}{2} \pi_{inv}^{NI} \leq \nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI})
\]

\[
\Leftrightarrow \Delta_{inv} \leq \frac{2 \eta}{v} .
\]  

(27)

The above condition is a sufficient condition to have an equilibrium with full integration. To observe this, assume that condition (27) holds and we consider the following strategies. \(C_i\) chooses \(p_i = 0\). As before, we denote \(\nabla \pi_2 (0, e^{SI}) - c(e^{SI})\) by \(A\) and \(\nabla \pi_2 (e^{NI}, e^{NI}) - c(e^{NI})\) by \(B\). In addition, we consider the integration strategies \(\left( \left( int_1^I(p), int_1^I(p) \right), \left( int_2^I(p), int_2^I(p) \right) \right)\) and the effort strategies given by (23) and (24), respectively.

As shown in the proof of lemma 3 (see Claim 1 in the proof), the integration strategies and effort strategies are Nash equilibrium strategies in the subgame induced by the price profile \(p\). We have to show that \(p_1 = 0\) and \(p_2 = 0\) are Nash equilibrium price strategies by the customer. To observe this, suppose that \(C_1\) offers \(p_1 = 0\). By increasing \(p_2 \geq B\), \(C_2\) can receive a payoff of \(\nabla \pi_2 (e^{SI}, 0) + \frac{v}{2} \pi_{inv}^{SI} - p_2\), which can never be higher than its current payoff from no integration \(\frac{v}{2} \pi_{inv}^{NI}\), as condition (27) holds. Hence, the abovementioned strategies are indeed Nash equilibrium strategies.  

\[\Box\]
Proof of Lemma 5:

Proof. Without loss of generality, we assume that RU_2 is integrated with C_2, and RU_1 and C_1 are not integrated in semi-integration. If an equilibrium with semi-integration exists, then RU_1 must not integrate at $p_{\min}$. The payoff of the non-integrated research unit is $A = v\pi_2 (0, e^{SI}) - c(e^{SI})$. Hence, we must have $p_{\min} \leq A$. However, there are two possibilities in which we can observe semi-integration. First, both research units are willing to integrate at $p_{\max}$, and second, RU_2 is integrated at $p_{\max}$ but RU_1 does not integrate at $p_{\max}$. We assume that $p_{\min} \leq A$ and analyze the two cases as follows.

Case 1: Both research units are willing to integrate at $p_{\max}$. When RU_2 is integrated, RU_1 receives $\frac{1}{2}p_{\max} + \frac{1}{2}A$ by integrating (RU_1 is matched with the customer offering $p_{\max}$ with $\frac{1}{2}$ probability) and it receives $A$ by not integrating. Hence, in this case, we must have $p_{\max} \geq A$.

Case 2: RU_2 is willing to integrate at $p_{\max}$, but RU_1 is not. Comparing RU_1’s payoff from integration and no integration (when RU_2 is integrated), we observe that $p_{\max} \leq A$. Similarly, comparing RU_2’s payoff from integration and no integration (when RU_1 is not integrated), we observe that $p_{\max} \geq B = v\pi_2 (e^{NI}, e^{NI}) - c(e^{NI})$. Hence, in this case, we must have $p_{\max} \in [B, A]$.

Next, we consider the customers’ optimal price responses. For given $p_1$, we consider the optimal response of RU_2.

If $p_1 < B$, C_2 receives $\frac{v}{2}\pi_1^{NI}$ by offering $p_2 < B$. Moreover, if it offers $p_2 \geq B$, one of the research units is integrated while the other is not. Therefore, by offering $p_2 \geq B$, C_2 receives $v\pi_2 (e^{SI}, 0) + \frac{v}{2}\pi_1^{SI} - p_2$, which is decreasing in $p_2$. Hence, when $p_1 < B$, the optimal response of C_2 is $B$ if $B \leq v\pi_2 (e^{SI}, 0) + \frac{v}{2}\pi_1^{SI} - \frac{v}{2}\pi_1^{NI}$, and any $p_2 < B$ if $B > v\pi_2 (e^{SI}, 0) + \frac{v}{2}\pi_1^{SI} - \frac{v}{2}\pi_1^{NI}$.

If $p_1 \in [B, A]$, C_2 receives $\frac{v}{2}\pi_1^{SI}$ by offering $p_2 < p_1$ (as only one research unit is integrated with C_1 in that case). By offering $p_2 \geq p_1$, C_2 receives $v\pi_2 (e^{SI}, 0) + \frac{v}{2}\pi_1^{SI} - p_2$, which is always less than $\frac{v}{2}\pi_1^{SI}$ for all $p_2 \geq B$. This is because $v\pi_2 (e^{SI}, 0) \leq v\pi_2 (e^{NI}, 0) \leq v\pi_2 (e^{NI}, e^{NI}) - c(e^{NI}) = B$. Hence, when $p_1 \in [B, A]$, the optimal response of C_2 is any $p_2 < p_1$.

Finally, if $p_1 > A$, C_2 receives $\frac{v}{2}\pi_1^{SI}$ by offering $p_2 < A$ (as only one research unit is integrated with C_1 in that case). By offering $p_2 \geq A$, C_2 receives $v\pi_2 (0, 0) + \frac{v}{2}\pi_1^{SI} - p_2$, which is always less than $\frac{v}{2}\pi_1^{SI}$ for all $p_2 \geq A$. Hence, when $p_1 > A$, the optimal response of C_2 is any $p_2 < A$.

The optimal response of C_1 for a given price $p_2$ would also be symmetric. It is evident that in no circumstances would any customer offer a price as high as $A$. Thus, case 1 depicted above, in which $p_{\max} \geq A$, is never realized in equilibrium. Therefore, if we observe semi-integration in equilibrium, case 2 must hold, in which we have $p_{\max} \in [B, A]$. From the optimal response functions, we observe that such a possibility
can occur only if $B \leq \pi_2(e^{SI}, 0) + \frac{\nu}{2} \pi_{inv}^{SI} - \frac{\nu}{2} \pi_{inv}^{NI}$, in which case, $p_{max} = B$ and $p_{min} < p_{max}$. Hence, a necessary condition to have an equilibrium with semi-integration is

$$\bar{\pi}_2(e^{SI}, 0) + \frac{\nu}{2} \pi_{inv}^{SI} - \frac{\nu}{2} \pi_{inv}^{NI} \geq \bar{\pi}_2(e^{NI}, e^{NI}) - c(e^{NI}) \iff \Delta_{inv} \geq \frac{2\eta}{\nu}. \quad (28)$$

The above condition is also a sufficient condition to have full integration in equilibrium. To observe this, assume that (28) holds and we denote $\bar{\pi}_2(0, e^{SI}) - c(e^{SI})$ by $A$ and $\bar{\pi}_2(e^{NI}, e^{NI}) - c(e^{NI})$ by $B$. Consider the following strategies. $C_1$ chooses $p_1 = 0$ and $C_2$ chooses $p_2 = B$. Consider the integration strategies $(int_1^I(p), int_1^S(p))$, $(int_2^I(p), int_2^S(p))$ and the effort strategies (23) and (24), respectively.

As shown in the proof of lemma 3 (see Claim 1 in the proof), the integration strategies and effort strategies are the Nash equilibrium strategies in the subgame induced by $p$. From our derivation of the optimal response functions above, we observe that $p_1 = 0$ and $p_2 = B$ are Nash equilibrium price strategies when condition (28) holds. Hence, the abovementioned strategies are indeed Nash equilibrium strategies.

**Proof of Lemma 6:**

**Proof.** Note that $e^{NI}$ satisfies (4) and $e^w$ satisfies (10). Denote $(\nu + \gamma) \left(1 - q(e^{NI})\right)$ and $\nu \left(1 - \frac{q(e^{NI})}{2}\right)$ by $A$ and $B$, respectively. A direct comparison of $A$ and $B$ shows that $A \leq B$ if and only if $\frac{2\nu}{\nu + 2\gamma} \leq q(e^{NI})$. By (4), $B = \frac{c'(e^{NI})}{q'(e^{NI})}$. Therefore,

$$A \leq B \iff (\nu + \gamma) \left(1 - q(e^{NI})\right) \leq \frac{c'(e^{NI})}{q'(e^{NI})} \iff (\nu + \gamma) \left(1 - q(e^{NI})\right) q'(e^{NI}) - c' \leq 0 \iff e^w \leq e^{NI} \text{ (by Assumption 3).}$$

**Proof of Proposition 3:**

**Proof.** From (7), (8), and (12), we observe that a no-integration equilibrium is inefficient if

$$\frac{(c(e^{SI}) - 2c(e^{NI}))}{\nu + \gamma} \leq \Delta_{inv} \leq \frac{2\eta}{\nu}. \quad (29)$$
and a semi-integration equilibrium is inefficient if

\[
\frac{2\eta}{\nu} \leq \Delta_{\text{inv}} \leq \frac{c(e^{\text{SI}}) - 2c(e^{\text{NI}})}{\bar{\nu} + \nu}.
\]

(30)

It can be shown by example that (29) is not vacuous. Below, we show that (30) cannot hold. In particular, we show that whenever \( W^{\text{SI}} \leq W^{\text{NI}} \), or equivalently, the right-hand side inequality in (30), holds, we cannot observe a semi-integration arrangement in equilibrium, or equivalently, the left-side inequality cannot hold. To observe this, it is useful to start with the following claim:

\[
2\nu \pi_2^2(e^{\text{SI}}, 0) \leq \pi_{\text{inv}}^{\text{SI}} - c(e^{\text{SI}}).
\]

(31)

The claimed relationship in (31) can be derived as follows:

\[
\nu \pi_2(e^{\text{SI}}, 0) \leq \nu \pi_2(0, 0) = \nu \pi_1(0, 0) \leq \nu \pi_1(e^{\text{SI}}, 0) - c(e^{\text{SI}}),
\]

\[\Leftrightarrow \nu \pi_2(e^{\text{SI}}, 0) + \nu \pi_2(e^{\text{SI}}, 0) \leq \nu \pi_2(e^{\text{SI}}, 0) + \nu \pi_1(e^{\text{SI}}, 0) - c(e^{\text{SI}}),\]

\[\Leftrightarrow 2\nu \pi_2(e^{\text{SI}}, 0) \leq \nu \pi_{\text{inv}}^{\text{SI}} - c(e^{\text{SI}})\]

We rewrite (31) as \(2\nu \pi_2(e^{\text{SI}}, 0) + \nu \pi_{\text{inv}}^{\text{SI}} \leq \nu \pi_{\text{inv}}^{\text{SI}} + \nu \pi_{\text{inv}}^{\text{SI}} - c(e^{\text{SI}}) = W^{\text{SI}}\). If a semi-integration arrangement is inefficient, then \( W^{\text{SI}} \leq W^{\text{NI}} \), and therefore,

\[
2\nu \pi_2(e^{\text{SI}}, 0) + \nu \pi_{\text{inv}}^{\text{SI}} \leq W^{\text{NI}} = (\bar{\nu} + \nu) \pi_{\text{inv}}^{\text{NI}} - 2c(e^{\text{NI}}).
\]

After rearranging terms, we obtain

\[
\nu \pi_{\text{inv}}^{\text{SI}} - \nu \pi_{\text{inv}}^{\text{NI}} \leq \nu \pi_{\text{inv}}^{\text{NI}} - 2c(e^{\text{NI}}) - 2\nu \pi_2(e^{\text{SI}}, 0),
\]

\[\Leftrightarrow \frac{\nu}{2} (\pi_{\text{inv}}^{\text{SI}} - \pi_{\text{inv}}^{\text{NI}}) \leq \nu \pi_2(e^{\text{NI}}, e^{\text{NI}}) - 2c(e^{\text{NI}}) - 2\nu \pi_2(e^{\text{SI}}, 0),
\]

\[\Leftrightarrow \frac{\nu}{2} \Delta_{\text{inv}} \leq \eta,
\]

which implies that we cannot observe semi-integration in equilibrium.