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Economies of scope, relatedness, and the dynamics of corporate diversification

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Abstract

The idea that relatedness between businesses enhances the tendency for diversifying firms to persistently combine those businesses dominated corporate diversification research. This study uses a dynamic model to qualify that proposition. The model considers alternative types of economies of scope and demonstrates that, with inter-temporal economies from redeployment of tangible resources, the effect of relatedness on diversification is very distinct from the assumed relationship. In particular, moderate rather than the strongest relatedness involved in inter-temporal economies creates the strongest diversification propensity. Moreover, the effect of relatedness on that propensity depends on other determinants of economies of scope. The study develops hypotheses for those complex relationships and suggests empirical operationalizations, encouraging empiricists to retest the implications of relatedness for the dynamics of corporate diversification.
ECONOMIES OF SCOPE, RESOURCE RELATEDNESS, AND THE DYNAMICS OF CORPORATE DIVERSIFICATION

INTRODUCTION

A key rationale for corporate diversification is that firms aim for economies of scope (Panzar and Willig, 1981; Teece, 1980). From the resource-based view (Penrose, 1959), such economies represent a reduction in costs for a firm deploying resources in multiple businesses relative to the costs those businesses incur if managed as independent firms. The resource-based economies were linked to resource relatedness, similarity of resource requirements between businesses (Rumelt, 1974). Relatedness supports economies by enhancing the applicability of resources across combined businesses and enabling the frugal use of the resources (plants, employees, and technological and marketing knowledge) across those businesses (Hill, Hitt, and Hoskisson, 1992). Many empirical studies have confirmed that diversifying firms are more likely to enter more-related businesses (Neffke and Henning, 2013; Silverman, 1999; Wu, 2013).

Despite the compelling evidence that firms initiate diversification by entering related businesses, the evidence on the implications of relatedness for the resulting dynamics of corporate diversification has been mixed. On one hand, relatedness was argued to preserve combinations of businesses in the corporate scope (Bryce and Winter, 2009; Teece et al., 1994; Lien and Klein, 2013). That view was supported in studies verifying that firms are more likely to combine more-related businesses (Breschi, Lissoni, and Malerba, 2003; Fan and Lang, 2000; Lemelin, 1982). Moreover, relatedness of a business to other businesses in the firm scope was shown to reduce the chances of exiting that business (Lien and Klein, 2013; O’Brien and Folta, 2009). On the other hand, relatedness of the combined businesses can destabilize the scope.

1 While recognizing importance of demand-side economies of scope (Ye, Priem, and Alshwer, 2012) due to increased consumer willingness to pay for two products bundled by the same firm, this study focuses on the resource-side economies.
As Helfat and Eisenhardt (2004) argued, relatedness enables a firm to exit one business and enter another business, making the firm focused rather than diversified. Lee, Folta, and Lieberman (2010) illustrated that relatedness of an entered business to a firm’s other businesses raises the chance that the firm will exit the entered business. The opposing empirical findings introduced the ambiguity about the ultimate effect of relatedness on the probability that a firm will diversify into a pair of businesses. Besides the ambiguity about the ultimate effect of relatedness on the probability of diversification, two other issues listed in the first column of Table 1 were unresolved. Notably, the issues of whether relatedness alone suffices to predict the probability of diversification and how that probability evolves over time were not clear. The lack of clear answers to the questions in the first column of Table 1 resulted from three respective limitations in previous research mentioned in the second column of the same table.

Insert Table 1 here

The first flaw was that the previous views did not carefully discriminate the effects of relatedness on diversification between distinct types of economies of scope. That approach contrasted with Helfat and Eisenhardt (2004) who clarified that ‘inter–temporal’ economies are different from ‘intra–temporal’ economies. While intra–temporal economies occur when a firm shares its resources between businesses, inter–temporal economies emerge when a firm exits one business and enters another business by redeploying resources between them. The restrictive focus on economies from sharing resources between related businesses led to the strong belief that relatedness between two businesses should enhance the probability that a firm diversifies into them (Bryce and Winter, 2009; Teece et al., 1994), inspiring respective empirical tests (Breschi et al., 2003; Fan and Lang, 2000; Lemelin, 1982).
The second shortcoming was that the extant accounts implicitly assumed that the effect of relatedness on corporate diversification was independent of other determinants of economies of scope. However, as Penrose (1959) argued, economies in corporate diversification depend on the interaction of relatedness with ‘inducements,’ the return advantages of one business over another. Such advantages represent an opportunity cost of the continued use of resources in a business performing worse than an alternative business. Sakhartov and Folta (2015) derived that inducements moderate the effect of relatedness on economies of scope. When economies of scope determine corporate diversification decisions, inducements can also modify the effect of relatedness on the propensity of a firm to diversify.

The third limitation in the existing research was that theory about how relatedness affects the dynamics of corporate diversification was informal. Nevertheless, verbal arguments can be ‘very misleading’ in the complex settings with inter–temporal links between a firm’s choices (Ghemawat and Cassiman, 2007: 530). In the context of corporate diversification, such links are present between corporate scope decisions because resource redeployments are costly to reverse. As a result, a firm entering (or exiting) a business considers not only the current redeployment costs but also the costs of a possible future exit from (or re-entry into) that business, making corporate diversification path–dependent (Sakhartov and Folta, 2014). The informal reasoning in that complex context included the tenuous assumption that, in contrast to relatedness linked to resource sharing, relatedness linked to resource redeployment may only have short-run implications for corporate diversification (Bryce and Winter, 2009).

Considering those limitations, this study develops a dynamic model of corporate diversification. Rooted in the general principle of dynamic optimality (Bellman, 1957), the model uses the simulation-based technique resolving the challenges of the informal reasoning in
the complex corporate context. The model derives several novel insights listed in the last column of Table 1. First, the model identifies how the effect of relatedness on the probability of diversification depends on the type of economies of scope. With intra–temporal economies, the diversification propensity is enhanced by relatedness. In contrast, with inter–temporal economies, the diversification propensity has an inverted U–shape relationship with relatedness: firms are most likely to persist with combinations of businesses having intermediate rather than the highest levels of relatedness. Second, the model indicates that inducements critically moderate the effect of relatedness on the diversification propensity. Finally, relatedness involved in inter–temporal economies is shown to have very durable implications for diversification.

The delivered results make three important contributions to corporate diversification research. First, the revealed difference in the effect of relatedness on diversification between intra–temporal and inter–temporal economies motivates researchers to unmerge those effects in empirical operationalizations and reexamine the often-tested relationship between relatedness and diversification. Second, the study is first to rigorously derive the curvilinear relationship between the diversification propensity and relatedness present with inter–temporal economies, replacing the informal argument that firms persist with sets of more-related businesses. Third, the insight that inducements moderate the effect of relatedness on the diversification propensity necessitates the use of the interactions in empirical models predicting the probability and the direction of corporate diversification. This paper develops a set of hypotheses to let the identification of interactions between the previously-known determinants of scope economies.

DETERMINANTS OF ECONOMIES OF SCOPE

Two recent elaborations highlight determinants of economies of scope. First, Helfat and Eisenhardt (2004) explained that, in addition to ‘intra–temporal’ economies, firms use ‘inter–
temporal’ economies. While the former occur when a firm contemporaneously shares resources between its businesses, the latter are enacted when a firm withdraws all or part of its resources from one business and redeploy them to another business. Second, Levinthal and Wu (2010) clarified that intra–temporal economies involve ‘scale free’ intangible resources with no physical substance (technological and marketing knowledge). Such resources can be leveraged to new uses without withdrawing them from the original use. Conversely, inter–temporal economies involve ‘non–scale free’ tangible resources having constraints on their capacity (employees and manufacturing plants) and demanding the withdrawal from the original use to be redeployed elsewhere. With both economies relatedness, the similarity of resource requirements between businesses (Rumelt, 1974), was identified as the key determinant (Hill et al., 1992).

**Relatedness and intra–temporal economies of scope**

With intra–temporal economies, relatedness between businesses in the corporate scope promotes the applicability of the scale free knowledge across them (Bryce and Winter, 2009). A diversified firm can apply knowledge created in one business to another business, avoiding the costly duplication in the development of that knowledge (Teece, 1980). While the cross-applicability of knowledge hinges upon relatedness, there is no reason to assume that any level of relatedness results in positive economies. Because the knowledge transfer to receiving business units is costly (Maritan and Brush, 2003), such transfer costs can exceed small cost savings in unrelated diversification turning economies into diseconomies. Conversely, in related diversification, the costs of the knowledge transfer are likely to fall behind strong economies.

**Relatedness and inter–temporal economies of scope**

With inter–temporal economies, relatedness promotes the cross-applicability of non–scale free resources. That enhanced cross–applicability leads to the following two effects. On one hand,
relatedness reduces the current costs of redeploying non–scale free resources to the new business incurred in entering that business (Montgomery and Wernerfelt, 1988). On the other hand, relatedness enhances the reversibility of redeployment (or non-redeployment) by reducing the costs of a possible future reversal of that decision (Sakhartov and Folta, 2015).

In addition to redeployment costs, economies from resource redeployment depend on ‘inducements,’ return advantages of one business over another (Penrose, 1959). Sakhartov and Folta (2015) summarize three dimensions of inducements. The first dimension is the current return advantages in the new business (Silverman, 1999; Wu, 2013). The second dimension is return volatilities (Kogut and Kulatilaka, 1994; Triantis and Hodder, 1990) in the current and the new businesses. Finally, the third dimension is return correlation between the current and the alternative businesses (Triantis and Hodder, 1990). The three dimensions capture the relative attractiveness of the alternative business for resource redeployment. Sakhartov and Folta (2015) derived that the effect of relatedness on inter–temporal economies is moderated by inducements and cannot be identified without such interactions.

**Implications of relatedness for corporate scope**

Based on the suggested impacts of relatedness on the value created in corporate diversification, multiple exploratory studies assessed the relation between relatedness of a pair of businesses and the probability that a firm will diversify into that pair. For instance, Lemelin (1982) examined diversification patterns of more than 2,000 Canadian firms in early 1970s. The study found that the probability of diversification in a pair of businesses is positively associated with relatedness measured as the similarity of distribution systems in those businesses based on input–output tables. Fan and Lang (2000) studied the probability of diversification into a pair of businesses from nearly 500 U.S. industries in years 1982, 1987, and 1992. They reported that diversifying
firms are more likely to own more–related segments, with relatedness captured as the affiliation with the same two–digit U.S. Standard Industry Classification (SIC) code. That result was robustly confirmed in the same study with relatedness measured as the similarity in both inputs and outputs between the segment industries based on input–output tables. Finally, Breschi et al. (2003) explored the patterns of technological diversification in a large sample of firms in Europe and the U.S. in years between 1982 and 1993. That study demonstrated that a firm active in one technological field is more likely to participate in another field when the two fields are more–related, with relatedness measured as co–occurrence of the two technological fields in patents.

Despite some empirical evidence on the impact of relatedness on diversification patterns, the tests were based on the underdeveloped theory. In particular, the theory built off the assumption that the positive effect of relatedness on economies of scope translates into an unconditional positive effect of relatedness on the corporate scope. Notably, Teece et al. (1994: 5) and Lien and Klein (2013: 1480) assume that activities which are more related will be more frequently combined within the same corporation. Bryce and Winter (2009: 1573) explained that intra–temporal economies from sharing scale free knowledge between related businesses preserve corporate diversification into those businesses for a long time. In contrast, inter–temporal economies from redeploying non–scale free resources between related businesses were assumed to have only short–run implications (Bryce and Winter 2009: 1573).

To sum up, the existing theory and its empirical tests had three features illustrated in the third column of Table 1. First, the theory assumed, rather than derived, that the probability of diversification is strongly positively associated with relatedness. Second, the effect of relatedness between businesses on the propensity of a firm to diversify into those businesses was implicitly assumed independent of other determinants of economies of scope. Finally, the implications of
relatedness with inter–temporal economies were deemed unimportant in the long run. The next section builds the model free of those tenuous assumptions.

**DYNAMIC MODEL OF CORPORATE DIVERSIFICATION**

To scrutinize the implications of relatedness for corporate diversification, this section builds a dynamic model. The approach, rooted in the principle of dynamic optimality (Bellman, 1957), was common to rationalize investors’ lifetime allocation of wealth across a portfolio of securities (Merton, 1969) and was extended to corporate diversification (Kogut and Kulatilaka, 1994; Sakhartov and Folta, 2014; 2015; and Triantis and Hodder, 1990). Resembling an investor keeping all her wealth in a single security, the firm in the model is initially focused on one business deploying all its resources in product market $i$. As an investor can allocate her cash to another security, so can the firm use its resources in an alternative product market $j$. Specifically, scale free resources can be shared between $i$ and $j$. Non–scale free resources may be fully or partially withdrawn from $i$ and redeployed to $j$, or vice versa, at any time $t$ before the end of the resource lifecycle $t = T$. Proportions $m_i$ and $m_j = 1 - m_i$ of non–scale free resources used respectively in markets $i$ and $j$ at time $t$ represent corporate diversification choices, with $m_i$ serving as a single control variable. Like a sequence of investment decisions by an individual investor in the portfolio choice model, the chain $\{m_i\}_{t=0}^{T-1}$ of corporate diversification choices represents the policy function in this model.

The corporate context in the model involves the three dimensions of inducements: the current return advantage, return volatilities, and return correlation. Relatedness is modeled using its two ramifications, the ease of sharing of scale free knowledge and the costs of redeployment of non–scale free resources. Redeployment costs make diversification choices path–dependent
(payoff to future choices depend on current choices) and, hence, intractable not only qualitatively (Ghemawat and Cassiman, 2007) but also analytically (Haugh and Kogan, 2007). The structural similarity of the dynamic allocation of corporate resources to the dynamic portfolio choice model (Merton, 1969) and the generality of the principle of dynamic optimality (Bellman, 1957) enable the use of a numerical method originally developed to optimize the dynamic portfolio choices. In particular, the intractability is resolved with the simulation–based technique of Brandt et al. (2005) illustrated in van Binsbergen and Brandt (2007).

To use the method of Brandt et al. (2005), the model represents returns $C_{it}$ and $C_{jt}$ in product markets $i$ and $j$ at time $t$ as two random state variables evolving in discrete time. The model simulates a large number of paths for $C_{it}$ and $C_{jt}$, with realizations $C_{kit}$ and $C_{kjt}$ on path $k$ at time $t$. Accordingly, proportions $m_{kit}$ and $m_{kjt} = 1 - m_{kit}$ capture a diversification choice on path $k$ at time $t$. The sequence $\{m_{kit}\}_{i=0}^{T-1}$ of diversification choices on path $k$ is identified by using the dynamic programming approach (Bellman, 1957). Namely, starting with time $t = T - 1$ when a diversification choice can be made the last time before the end of the resource lifecycle, the model finds on each path proportion $m_{kit}^*$ that maximizes a Taylor approximation of the value function, the utility of the value the firm accumulates over the lifecycle of its resources by allocating those resources between the two businesses. Like in van Binsbergen and Brandt (2007), the coefficients for the Taylor series are estimated by running the ordinary least square regression of the value function realized at $t = T$ on a polynomial of the state variables at time $t = T - 1$. The algorithm then proceeds recursively from $t = T - 1$ to $t = 0$. Key elements of the model, the corporate context and the identification of diversification choices, are elaborated below.
Corporate context

In the model, at every time $t$ the firm seeks to maximize the value function $U_t$ of the terminal value $V_T$ accumulated through resource deployment choices $\{m_s\}_{s=t}^{T-1}$ undertaken over the remaining lifecycle of the firm’s resources. Formally, the problem the firm faces is:

$$U_t = \max_{\{m_s\}_{s=t}^{T-1}} E_t[u(V_T)], \quad (1)$$

In Equation 1, $E_t[\cdot]$ is the expectation based on the information available at time $t$. The function $u(\cdot)$ captures the utility of the risk–averse firm from having value $V_T$ given the risk attached to that value. The property of risk–aversion is common for individuals optimizing the allocation of wealth across risky securities. Concave utility functions have also been repeatedly used in the literature (e.g., Asplund, 2002; Carceles Poveda, 2003) to specify the tradeoff between risk and returns faced by firms. The model applies the prevalently used utility function with constant absolute risk aversion (Arrow, 1971):

$$u(V_T) = 1 - e^{-\gamma V_T}, \quad (2)$$

where $\gamma$ is the coefficient capturing risk aversion. That parameter has an intuitive interpretation that its higher value implies that the firm puts a greater discount for the risk associated with $V_T$.

Like van Binsbergen and Brandt (2007), the model specifies the state variables $C_{it}$ and $C_{jt}$ as a vector autoregression with one lag VAR(1). That stochastic process captures linear interdependence between univariate autoregressions AR(1) often used to capture the evolution of a random variable. Formally,

$$\begin{bmatrix} C_{it} \\ C_{jt} \end{bmatrix} = \begin{bmatrix} A_i \\ A_j \end{bmatrix} + \begin{bmatrix} \mu_i \\ 0 \end{bmatrix} C_{it-1} + \begin{bmatrix} 0 \\ \mu_j \end{bmatrix} C_{jt-1} + \begin{bmatrix} \epsilon_{it} \\ \epsilon_{jt} \end{bmatrix}. \quad (3)$$

Equation 3 involves the dimensions of inducements used in the literature. Thus, intercepts $A_i$ and
\(A_j\) capture the current return advantage \((A_j - A_i)/A_i\). Errors \(\epsilon_{it}\) and \(\epsilon_{jt}\) have variances \(\sigma_i^2\) and \(\sigma_j^2\) and correlation \(\rho\) capturing return volatilities and correlation, respectively. With Equation 3, \(C_{it}\) and \(C_{jt}\) are realized in \(T + 1\) points in time \(t \in [0, T]\). In the context void of economies of scope, an arbitrary realization \(k\) of returns \(C_{kit}\) and \(C_{kjt}\) and deployment of proportions \(m_{kit}\) and \((1 - m_{kit})\) of resources in markets \(i\) and \(j\), respectively, would generate the net return at time \(t\):

\[
F_{kt} = [m_{kit}C_{kit} + (1 - m_{kit})C_{kjt}]. \tag{4}
\]

Relatedness is modeled using its known impacts on the two types of economies of scope. With inter–temporal economies, relatedness reduces redeployment costs. Such costs represent a loss in efficiency present in withdrawing non–scale free resources from another business and redeploying them to a focal business relative to returns earned when the resources had been originally used in the focal business (Montgomery and Wernerfelt, 1988). Because the model captures efficiency with market returns, total costs of redeployment of resources to market \(j\) (\(i\)) at time \(t\) are modeled as a product of (a) the marginal redeployment cost \(S\) of a unit of resources; (b) the amount \(\max[0, m_{kit-1} - m_{kit}] \ (\max[0, m_{kit} - m_{kit-1}])\) of resources redeployed to market \(j\) (\(i\)) between \(t - 1\) and \(t\); and (c) average returns \(\hat{C}_{jt}\) (\(\hat{C}_{it}\)) at time \(t\) in the recipient business. That specification of redeployment costs has precedents: Sakhartov and Folta (2014; 2015) modeled redeployment costs as a proportion of returns in the receiving unit. Kogut and Kulatilaka (1994) also modeled switching costs as a proportion of value outcomes, although in their model such outcomes are captured with average production costs. Thus, with inter–temporal economies, the net return earned by the firm at time \(t\) is

\[
F_{kt} = [m_{kit}C_{kit} + (1 - m_{kit})C_{kjt}] - S \{\max[0, m_{kit} - m_{kit-1}]\hat{C}_{it} + \max[0, m_{kit-1} - m_{kit}]\hat{C}_{jt}\}. \tag{5}
\]
With intra–temporal economies, relatedness increases the applicability of scale free knowledge between businesses (Bryce and Winter, 2009). Therefore, the firm can use knowledge from one business in another business, thereby reducing total costs (Teece, 1980) and increasing net returns. Small cost savings in unrelated diversification can be exceeded by the costs of transferring the knowledge to the receiving business (Maritan and Brush, 2003), turning economies into diseconomies. Alternatively, large cost savings in related diversification surpass the costs of the knowledge transfer, creating positive economies. Those two polar scenarios for intra–temporal economies can be represented as a continuum amending Equation 5 as follows

\[
F_{it} = \left[ m_{kit} C_{kit} + (1 - m_{kit}) C_{kj} \right] - S \left\{ \max\{0, m_{kit} - m_{kit-1}\} \hat{C}_{it} + \max\{0, m_{kit-1} - m_{kit} \hat{C}_{jt}\} \right\} + I(\beta - 1) (m_{kit} \hat{C}_{it} + (1 - m_{kit}) \hat{C}_{jt})
\]

(6)

In Equation 6, \( I \) is an indicator of knowledge sharing: \( I = 1 \) if \( 0 < m_{kit} < 1 \), and \( I = 0 \) otherwise. Coefficient \( \beta \) is the sharing factor directly capturing the effect of relatedness on intra–temporal economies. When \( \beta < 1 \), \( i \) and \( j \) are weakly related and diversification creates diseconomies. When \( \beta = 1 \), \( i \) and \( j \) are moderately related and the firm generates neither economies nor diseconomies. When \( \beta > 1 \), \( i \) and \( j \) are strongly related and diversification creates economies.

The specification of relatedness in intra–temporal economies has precedents. In particular, Sakhartov and Folta (2014) also modeled intra–temporal economies with the sharing factor.

The representation of relatedness imposes no dependence structure between \( S \) and \( \beta \). Although a strong negative relationship might be assumed because relatedness is defined as the general similarity of resource requirements between two businesses (Rumelt, 1974: 29), the model avoids assuming that strong negative relationship for two reasons. First, the extant theory does not argue that two businesses using similar scale free resources should necessarily rely on equally similar non–scale free resources. Second, by not imposing a specific structure on the
underexplored relationship, the model remains agnostic about the nature of that relationship. That approach enables the flexibility in examining the dynamic impact of relatedness on diversification for any dependence structure, instead of making the restrictive assumption.

The value dynamics in the context with intra–temporal and inter–temporal economies are shown in Table 2 for the case $T = 10$. The first row indicates how value $V_t$ accumulated by time $t$ evolves over time and how that value derives from the net return on investment $F_t$. To isolate economies of scope from the option to buy new resources, returns are not reinvested in markets $i$ and $j$ but are put into a risk–free account with interest rate $r$. The second row splits the total value $V_t$ accumulated over the resource lifecycle into the past (normal font) and the future (bold font) parts at time $t$. With the used utility, only the future part $\tilde{V}_t$ shown in the third row is relevant for the identification of diversification choices.

Insert Table 2 here

**Identification of corporate diversification choices**

Non–trivial redeployment costs ($S > 0$) make diversification path–dependent, precluding the analytical identification of the policy function (Haugh and Kogan, 2007). The intractability is resolved using the simulation–based technique of Brandt et al. (2005) relying on the principle of dynamic optimality (Bellman, 1957) and illustrated in van Binsbergen and Brandt (2007). Five steps in van Binsbergen and Brandt (2007) are amended with two last steps summarizing the diversification propensity. Moreover, the algorithm is replicated for numerous combinations of the determinants of economies of scope to identify how those determinants individually and jointly affect the diversification propensity.
• **Step 1.** Simulate \( n \) of paths for \( C_{it} \) and \( C_{jt} \) based on Equation 3. Discretize proportion \( m_{kt} \in [0, 1] \) of resources deployed in \( i \) with a grid \( m_{kt} \in \{0, 1/L, 2/L, ..., (L-1)/L, 1\} \), where \( L \) is a discretization number. Estimate mean returns \( \hat{C}_n \) and \( \hat{C}_{jt} \) for Equation 6.

• **Step 2.** Take path \( k \) and consider time \( t = T - 1 \) when returns are earned the last time. For a current diversification choice \( m_{kt-1} \) and a most recent choice \( m_{kt-2} \) taken from their discretized values, use Equation 6 and Table 2 to estimate value \( \hat{V}_{kt} \) relevant for the optimization of the diversification choice \( m_{kt-1} \). Repeat the estimation of \( \hat{V}_{kt} \) for each of \( n \) paths. Use Equation 2 to compute utility \( u(\hat{V}_{kt}) \). Assemble the dataset consisting of \( n \) combinations of \( u(\hat{V}_{kt}), C_{kt-1}, \) and \( C_{jt-1} \). Run an ordinary least square regression model such that \( u(\hat{V}_{kt}) = \beta_0 + \beta_1 C_{kt-1} + \beta_2 C_{jt-1} + \beta_3 C_{kt-1}^2 + \beta_4 C_{jt-1}^2 + \beta_5 m_{kt-1} \cdot C_{jt-1} + \psi^2 \).

• **Step 3.** Using the coefficients (\( \beta \)'s) estimated in Step 2, compute, on each path, the expected utility \( \hat{u}(\cdot) = E[u(\hat{V}_{kt}) | (C_{kt-1}, C_{jt-1}, m_{kt-1}, m_{kt-2})] \) conditioned on the firm being on path \( k \), having previously committed choice \( m_{kt-1} \), and committing choice \( m_{kt-2} \).

• **Step 4.** Repeat Steps 2 and 3 for all values of \( m_{kt-1} \) and \( m_{kt-2} \). Take values of \( \hat{u}(\cdot) \) calculated for a particular value of \( m_{kt-2} \) and, among them, select an optimal current choice \( m_{kt-1}^* | m_{kt-2} \) = \( \arg \max_{m_{kt-1}} \hat{u}(\cdot) \) conditioned on the considered most recent choice \( m_{kt-2} \). Repeat Step 4 for all values of the most recent diversification choice \( m_{kt-2} \).

• **Step 5.** Using the dynamic programming principle, proceed recursively backward from \( t = T - 2 \) to \( t = 0 \) with Steps 2, 3, and 4 and retrieve the matrix of diversification choices.

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2 Error \( \psi \) is normally distributed with zero mean.
(m^*_{kit} | m_{kit-1}) on all n paths conditioned on the most recent choices. In calculating value \( \vec{V}_{kt} \) relevant for the optimization of diversification choices, use Equation 6 and Table 2.

- **Step 6.** Given that the firm is initially focuses on market \( i \), proceed recursively forward from \( t = 0 \) to \( t = T - 1 \) through the conditional diversification choices \( (m^*_{kit} | m_{kit-1}) \) derived in Steps 5 and calculate the matrix of unconditional choices \( \{m^*_{kit}\}_{i=0}^{T-1} \). Each of n columns of that matrix represents the policy function on path \( k \). All columns together represent the policy function for all n states of the state variables \( C_{it} \) and \( C_{jt} \) over time.

- **Step 7.** Use the matrix of unconditional choices from Step 6 and estimate the probability \( p_t = \Pr([m^*_{kit} \neq 0 \land m^*_{kit} \neq 1]) \) of diversification at time \( t \) by dividing the number of cases where the firm is not fully focused on either \( i \) or \( j \) at time \( t \) by the total number of possible scenarios \( n \).

**RESULTS**

The analysis of the impact of relatedness on the dynamics of corporate diversification involves steps addressing the issues raised in Table 1. First, the diversification propensity is derived for various sets of the sharing factor and the marginal redeployment cost representing relatedness, while keeping other parameters constant. That design identifies separate and joint effects of the two roles of relatedness on the diversification propensity. Second, the diversification propensity is assessed for multiple combinations of the determinants of inter–temporal economies, when intra–temporal economies are disallowed. In that step, the interactions between inducements and redeployment costs in determining the diversification propensity are explicated. Finally, the evolution of the diversification propensity is traced over time, and the sensitivity of that
propensity to redeployment costs is checked at the end of the resource lifecycle. That analysis uncovers the long–run implications of relatedness with inter–temporal economies.

**Implications of relatedness for diversification propensity**

A key aim of the present study is to scrutinize the idea that relatedness unambiguously enhances the diversification propensity. Figure 1 illustrates the diversification propensity with the probability that the firm is diversified in the middle of the resource lifecycle. Inter–temporal economies of scope are represented with the marginal redeployment cost, varying continuously from its lowest possible value (the strongest relatedness) to a very high value (very weak relatedness). Intra–temporal economies are captured with the sharing factor, taking five discrete values. A very low factor (very weak relatedness) is shown with the broken line capturing very strong intra–temporal diseconomies of scope. A slightly–below–medium factor is shown with the line with downward–pointing triangles illustrating weak intra–temporal diseconomies. A moderate factor is revealed with the solid line capturing the lack of intra–temporal economies or diseconomies. A slightly–above–medium factor is shown with the line with upward–pointing triangles displaying weak intra–temporal economies. Finally, a very high factor (very strong relatedness) is demonstrated with the line with plus signs revealing very strong intra–temporal economies. Three patterns summarize the effects of relatedness on the diversification propensity.

First, Figure 1 reveals that the lines with a higher sharing factor are always located at least as high as the lines with a lower factor. The observed relative altitudes of the lines imply that, with intra–temporal economies, relatedness monotonously enhances the probability of diversification. That first result is not novel: it confirms the existing insight that relatedness of scale free knowledge between businesses increases the propensity of the firm to diversify into
those businesses. However, the magnitude of the divergence of the solid line from zero in Figure 1 is surprising. The strong divergence suggests that the firm is very likely to persistently diversify even when relatedness of knowledge between the combined businesses is mediocre, generating no intra–temporal economies of scope. In particular, when intra–temporal economies are absent, the probability of diversification can reach the value of 99 percent. Because of the model design, in the absence of intra–temporal economies, the diversification illustrated with the solid line in Figure 1 occurs due to inter–temporal economies of scope.

Second, most of the lines in Figure 1 have inverted U–shapes. That curvature suggests that, with inter–temporal economies, moderate rather than the strongest relatedness leads to the highest probability of corporate diversification. That result challenges the assumed generality of the monotonous positive relationship between relatedness and the diversification propensity.

Third, the lines in Figure 1 vary in concavity. Notably, the lines with moderate levels of the sharing factor are concave. Conversely, the lines with very low or very high factor are flat. With very high sharing factor on the lines with plus signs, the firm instantly redeployes part of resources to a new business to receive very strong intra–temporal economies. The firm then keeps diversifying regardless of redeployment costs. With very low sharing factor on the broken lines, the firm is reluctant to incur very strong diseconomies and therefore never redeployes resources to the new business regardless of redeployment costs. With mediocre sharing factor, the firm is indifferent to the trivial intra–temporal economies but is very sensitive to the magnitude of redeployment costs determining inter–temporal economies of scope. That third pattern in Figure 1 means that the effects relatedness has on corporate diversification with intra–temporal and inter–temporal economies are interdependent, necessitating the empirical operationalization of the interdependence in models predicting corporate diversification patterns.
Implications of inducements for diversification propensity

Although relatedness has been a primary focus for the research on corporate diversification, the implications of other determinants of economies of scope (inducements) for the dynamics of diversification should also be explicated. That clarification would be particularly necessary if the effect of relatedness on the diversification propensity depended on inducements. Whether such interdependences are present is analyzed below for each of the dimensions of inducements.

Current return advantage

Figure 2 presents the implications of the current return advantage for the probability of diversification. While the lowest diversification propensity corresponds to the strong current disadvantage, the figure fails to demonstrate a robust direct effect of the current advantage on corporate diversification. For example, with low redeployment costs, the strongest diversification propensity results from the zero current advantage. In turn, with high costs of redeployment, the positive current advantage leads to the strongest diversification propensity. Despite the unclear direct effect, the current return advantage systematically alters the effect of relatedness on the diversification propensity: with greater current advantages, a peak in that propensity shifts to higher redeployment costs.

Insert Figure 2 here

Return volatility

Figure 3 illustrates the effect of return volatility on the diversification propensity. The arms of a line in the figure are further apart from each other, the higher is volatility associated with that line. That pattern suggests that volatility substantially alter the effect of relatedness on the diversification propensity. In particular, volatility extends values of redeployment costs with which diversification is highly likely. Thus, the range of redeployment costs resulting in the
probability of diversification greater than 75 percent is six times broader with very high volatility than with low volatility.

Insert Figure 3 here

Return correlation

Figure 4 demonstrates the effect of return correlation on the diversification propensity. The arms of a line in the figure are more distant from each other, the more–negative is correlation on that line. That effect suggests that correlation modifies the effect of relatedness on the diversification propensity. Thus, the range of redeployment costs resulting in the probability of diversification greater than 75 percent is three times broader with strong negative correlation than with zero correlation.

Insert Figure 4 here

**Evolution of diversification propensity with different types of economies of scope**

The last candidate for the scrutiny of the effect of relatedness on diversification is the claim that, unlike intra–temporal economies, inter–temporal economies from redeploying non–scale free resources between related businesses has only short–run implications (Bryce and Winter, 2009: 1573). The model can test that claim by separately tuning relatedness involved in intra–temporal and inter–temporal economies of scope and tracing the probability of diversification over time.

Figure 5 reveals the long–run implications of relatedness. Panel A presents the effect of redeployment costs on the likelihood of diversification at the end of the resource lifecycle. Intra–temporal economies are disallowed. Relatedness involved in inter–temporal economies strongly affects the probability of diversification even at the end of the lifecycle. In particular, the inverted U–shape relationship robustly confirmed in Figures 1–4 persists by the end of the
lifecycle. Moreover, the magnitude of the relation continues to be very strong: the probability of diversification varies between zero and 99 percent depending on the value of redeployment costs.

Insert Figure 5 here

Panel B of Figure 5 depicts the dynamics of the probability of diversification when relatedness present in inter–temporal economies assumes a mediocre value. Three scenarios for intra–temporal economies are modeled: (a) economies are strong (the line with plus signs), (b) neither economies nor diseconomies are present (the solid line), and (c) diseconomies are strong (the broken line). The striking difference in the elevation between the line with plus signs and the broken line confirms that relatedness of scale free resources between businesses has a strong and long–lasting positive effect on diversification into those businesses (Bryce and Winter, 2009: 1573). However, the dynamics illustrated with the solid line, representing the context void of intra–temporal economies, contrast with the existing conception that inter–temporal economies of scope may only have a short–run effect on corporate diversification. In the absence of intra–temporal economies, moderate relatedness involved in inter–temporal economies makes the probability of diversification grow over time and then keeps it very high (96%) in the long run.

Summary of new theoretical results

The following hypotheses summarize the new theoretical results amending the existing theory.

H1: The propensity to diversify in two businesses has an inverted U–shape relationship with relatedness involved in inter–temporal economies of scope between those businesses.

H2: The inverted U–shape relationship between the propensity to diversify in two businesses and relatedness involved in inter–temporal economies of scope is strongest when relatedness involved in intra–temporal economies of scope is moderate.
H3: The inverted U–shape relationship between the propensity to diversify in two businesses and relatedness involved in inter–temporal economies of scope sifts to lower levels of such relatedness with stronger current return advantages.

H4: The inverted U–shape relationship between the propensity to diversify in two businesses and relatedness involved in inter–temporal economies of scope expands onto a broader range for such relatedness with higher return volatilities.

H5: The inverted U–shape relationship between the propensity to diversify in two businesses and relatedness involved in inter–temporal economies of scope expands onto a broader range for such relatedness with more–negative return correlation.

**TOWARDS EMPIRICAL IDENTIFICATION OF THEORETICAL RESULTS**

Empirical models seeking to test the implications of relatedness for the dynamics of corporate diversification, elaborated with Hypotheses 1–5, can take the following form:

\[
Y = \gamma_0 + \gamma_1 K + \gamma_2 \beta_{ij} + \gamma_3 S_{ij} + \gamma_4 \beta_{ij}^2 S_{ij}^2 + \gamma_5 (C_{ij} - C_{ii}) + \gamma_6 S_{ij} + \gamma_7 (C_{ij} - C_{ii})^2 + \gamma_8 \sigma_i + \gamma_9 \sigma_j S_{ij}^2 + \gamma_{10} \sigma_i + \gamma_{11} \sigma_j S_{ij}^2 + \gamma_{12} \rho_{ij} + \gamma_{13} \rho_{ij} S_{ij}^2 + \varepsilon
\]

The dependent variable \( Y \) may be captured as the probability of the co–occurrence of businesses \( i \) and \( j \) in the corporate scope or the duration of that combination. Businesses can be identified based on the U.S. SIC. All \( \gamma \)'s are estimated coefficients. By \( K \), a vector of determinants of the diversification propensity other than those used in this study is denoted. The sharing factor \( \beta_{ij} \) can be measured with the Euclidean distance between patent profiles of SIC industries created by Brian Silverman. That measure inversely captures the similarity of the scale free technological knowledge between industries. Redeployment costs \( S_{ij} \) can be measured as the Euclidian distance between industries \( i \) and \( j \) in non–scale free resources, estimated from industry
balance sheets (Compustat Segments). Current returns $C_{it}$ and $C_{jt}$ can be taken from the Compustat Segments as mean industry return on asset (ROA) at time $t$. Volatilities $\sigma_i$ and $\sigma_j$ can be computed as standard deviations of industry ROA. Return correlation $\rho_{ij}$ can be measured as correlation of mean ROA between industries. Finally, the distribution of the error term $\epsilon$ is chosen based on the used dependent variable, the probability of the co–occurrence or the duration of the co–occurrence.

Hypotheses H1–H5 are tested by checking the signs of the respective $\gamma$’s. Based on the theoretical results derived in the present study, the expected signs of the coefficients are the following: $\gamma_4 > 0$ (H2), $\gamma_6 < 0$ (H1), $\gamma_7 < 0$ (H3), $\gamma_9 < 0$ and $\gamma_{11} < 0$ (H4), and $\gamma_{13} > 0$ (H5).

**DISCUSSION**

Does relatedness between businesses enhance the tendency for a diversifying firm to persistently combine them? The existing theory answered that question affirmatively (Bryce and Winter, 2009; Lien and Klein, 2013; Teece et al., 1994). The direct relationship, inferred from the positive impact of relatedness on intra–temporal economies of scope from knowledge sharing, has been tested empirically (Breschi et al., 2003; Fan and Lang, 2000; Lemelin, 1982). However, there has also been recognition of inter–temporal economies of scope, with which relatedness can destabilize the corporate scope making firms exit some businesses to enter related businesses (Helfat and Eisenhardt, 2004; Lee et al. 2010; Penrose, 1959; Sakhartov and Folta, 2014). With that insight, the ultimate effect of relatedness on the dynamics of diversification is less certain than previously believed. In addition, determinants of economies of scope other than relatedness were argued to interact with relatedness in creating economies (Penrose, 1959; Sakhartov and Folta, 2015) and therefore can alter the effect of relatedness on the diversification propensity. To
improve the understanding of the effect of relatedness on the dynamics of diversification, this study builds the dynamic model of diversification choices including both types of economies of scope. The model delivers several stimulating insights for the corporate diversification research.

First, the model identifies separate and joint effects of relatedness on the diversification propensity with the two types of economies of scope. As known before, that propensity is enhanced by relatedness of scale free knowledge involved in intra–temporal economies. In contrast, with inter–temporal economies, the diversification propensity has an inverted U–shape relationship with relatedness. That means that firms are most likely to persist with pairs of businesses having intermediate rather than the highest relatedness of non–scale free resources. Moreover, the two effects of relatedness are interdependent. In particular, the diversification propensity is most sensitive to relatedness present in inter–temporal economies when relatedness involved in intra–temporal economies is moderate. The difference between the two effects of relatedness on corporate diversification, along with the interdependence between those effects, necessitates the separate operationalizations of the two manifestations of relatedness and the re–examination of the often–tested relationship between diversification and relatedness.

Second, relatedness alone does not suffice to predict the proclivity of a firm to diversify. Inducements significantly moderate the effect of relatedness. The derived interactions suggest that the effect of relatedness on diversification cannot be empirically identified, unless its interactions with inducements are captured. The study summarizes the empirical relationships and provides the direction for their empirical operationalization, laying the groundwork for a better empirical identification of the determinants of the diversification propensity.

Finally, the paper uncovers the dynamics of diversification based on inter–temporal economies involving non–scale free resources. In contrast to the extant view, the diversification
propensity remains very sensitive to relatedness of non–scale free resources in the long run. With moderate relatedness present in inter–temporal economies, the firm diversifies, even when intra–temporal economies from sharing scale free knowledge are absent. Moreover, such propensity grows rather than declines over time. Beyond revising the idea that relatedness of non–scale free resources may only have short–run effect on diversification (Bryce and Winter, 2009), the result warns about the limitations of the recently advocated measure of relatedness based on survival of business combinations (Bryce and Winter, 2009; Lien and Klein, 2013; Teece et al., 1994). When moderately–related businesses are at least as likely to be combined as strongly related businesses, inferring stronger relatedness from the co–occurrence of businesses in the corporate scope is tenuous.

**Limitations**

The present study builds the dynamic model of diversification choices to improve the understanding of how the dynamics of corporate diversification bear upon resource relatedness. Nevertheless, the used methodology has some intrinsic limitations. The generalizability of the results derived numerically may be compromised by arbitrary choices of the model’s parameters. The current study, like other studies using simulation, attempts to mitigate the concern about the generalizability by undertaking extensive sensitivity checks and confirming that the directions of all the reported relationships are robust in a wide variety of parameter specifications.

Some readers may find the used model of diversification too simplistic because it ignores time lags in resource allocation, organizational inertia, bounded rationality of corporate managers, competitive advantages of incumbent or new firms in entered businesses, and possible acquisitions and divestitures of resources. While adding those features would enrich the enquiry,
they would also considerably complicate the model making it intractable even numerically. Future research might try building more comprehensive models of corporate diversification.

**Conclusion**

The present paper scrutinizes the implications of resource relatedness for the dynamics of corporate diversification. The benchmark for the scrutiny is the very prevalent belief that firms are more likely to diversify in combinations of more–related businesses, because relatedness enhances intra–temporal economies of scope from contemporaneously sharing scale free knowledge between combined businesses. The study uses the dynamic model of diversification choices to reconsider that proposition. The model follows recent research and involves the effects relatedness has on the diversification propensity with various types of economies of scope. The model demonstrates that, with inter–temporal economies from redeployment of non–scale free resources, the effect of relatedness on the diversification propensity remarkably differs from what is commonly assumed. In particular, with inter–temporal economies, moderate rather than the strongest relatedness leads to the strongest diversification propensity. Moreover, the effect of relatedness involved in inter–temporal economies on the diversification propensity is critically moderated by other determinants of inter–temporal economies of scope and by relatedness involved in intra–temporal economies. The study develops empirically testable hypotheses for those complex relationships importantly qualifying the commonly tested, simple proposition that firms are more likely to diversify in more–related businesses. The paper also suggests empirical operationalizations for the developed hypotheses. Those developments can encourage empiricists to retest the dynamic implications of relatedness for corporate diversification.
REFERENCES


Figure 1: Implications of relatedness for the dynamics of corporate diversification

Figure 1 shows the probability $p_t$ of the firm being diversified in markets $i$ and $j$ in the middle of the resources lifecycle. The first (inverse) proxy for relatedness, the marginal redeployment cost $S$, represents inter–temporal economies of scope and varies within the interval $S \in [0.0, 1.3]$. In particular, $S = 0.0$ corresponds to the strongest possible relatedness between $i$ and $j$ in non–scale free resources, whereas $S = 1.3$ corresponds to very weak relatedness. In both panels, the second (direct) proxy for relatedness, the sharing factor $\beta$, represents intra–temporal economies of scope and takes five values $\beta \in \{0.90, 0.99, 1.00, 1.01, 1.0\}$. To clarify, with $\beta = 0.90$ (depicted with the broken lines), $i$ and $j$ are very weakly related in scale free knowledge, resulting in strong intra–temporal diseconomies of scope. With $\beta = 0.99$ (depicted with the lines with downward–pointing triangles), knowledge relatedness is slightly below average, resulting in weak diseconomies. With $\beta = 1.00$ (depicted with the solid lines), knowledge relatedness is average, generating neither diseconomies nor economies. With $\beta = 1.01$ (depicted with the lines with upward–pointing triangles), knowledge relatedness is slightly above average, creating weak economies. With $\beta = 1.10$ (depicted with the lines with plus signs), $i$ and $j$ are very strongly related in knowledge, resulting in strong economies. The following values of other parameters were used to generate the graphs: the length of the resource lifecycle, $T = 10$; the coefficient of absolute risk aversion, $\gamma = 0.5$; the discreteness with which resource capacity may be redeployed, $L = 10$; the offsets capturing the current returns, $A_i = A_j = 0.50$; the trends for returns, $\mu_i = \mu_j = 0.1$; the variances of the innovation terms capturing the volatilities of returns, $\sigma_i^2 = \sigma_j^2 = 0.45$; the correlation of the innovation terms capturing the correlation of returns, $\rho = 0$; the number of simulated paths for the returns, $n = 10,000$; the risk–free interest rate, $r = 0.1$; and the value of the invested resources, $V_0 = 1$. 
Figure 2. Implications of redeployment costs and the current return advantage for the dynamics of corporate diversification

Figure 2 shows the probability $p_t$ of the firm being diversified in markets $i$ and $j$ in the middle of the resources lifecycle. The first (inverse) proxy for relatedness, the marginal redeployment cost $S$, represents inter–temporal economies of scope and varies within the interval $S \in [0.0, 1.3]$. In particular, $S = 0.0$ corresponds to the strongest possible relatedness between $i$ and $j$ in non–scale free resources, whereas $S = 1.3$ corresponds to very weak relatedness. In both panels, the second proxy for relatedness, the sharing factor $\beta$, is set to the medium value $\beta = 1.00$ with which knowledge relatedness is mediocre and intra–temporal diseconomies or economies are absent. In both panels, the current return advantage $(A_i - A_j)/A_i$ takes five values. In particular, in the broken lines, $A_i = 0.50$ and $A_j = 0.40$ showing the strong negative current advantage. In the lines with downward–pointing triangles, $A_i = 0.50$ and $A_j = 0.45$ revealing the weak negative current return advantage. In the solid lines, $A_i = A_j = 0.50$ capturing the zero current advantage. In the lines with upward–pointing triangles, $A_i = 0.50$ and $A_j = 0.55$ showing the weak positive current advantage. In the lines with plus signs, $A_i = 0.50$ and $A_j = 0.60$ revealing the strong positive current advantage. The following values of other parameters were used to create the graphs: the length of the resource lifecycle, $T = 10$; the coefficient of absolute risk aversion, $\gamma = 0.5$; the discreteness with which resource capacity may be redeployed, $L = 10$; the trends for returns, $\mu_i = \mu_j = 0.1$; the variances of the innovation terms showing return volatilities, $\sigma_i^2 = \sigma_j^2 = 0.45$; the correlation of the innovation terms capturing return correlation, $\rho = 0$; the number of simulated paths for the returns, $n = 10,000$; the risk–free interest rate, $r = 0.1$; and the value of the invested resources, $V_0 = 1$.  

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Figure 3 shows the probability $p_i$ of the firm being diversified in markets $i$ and $j$ in the middle of the resources lifecycle. The first (inverse) proxy for relatedness, the marginal redeployment cost $S$, represents inter–temporal economies of scope and varies within the interval $S \in [0.0, 1.3]$. In particular, $S = 0.0$ corresponds to the strongest possible relatedness between $i$ and $j$ in non–scale free resources, whereas $S = 1.3$ corresponds to very weak relatedness. In both panels, the second proxy for relatedness, the sharing factor $\beta$, is set to the medium value $\beta = 1.00$ with which knowledge relatedness is mediocre and intra–temporal diseconomies or economies are absent.

In both panels, the variances $\sigma_i^2$ and $\sigma_j^2$ of the innovation terms capturing the volatilities of returns take five values. In particular, in the broken lines, $\sigma_i^2 = \sigma_j^2 = 0.05$ capturing very low volatility. In the lines with downward–pointing triangles, $\sigma_i^2 = \sigma_j^2 = 0.30$ revealing low volatility. In the solid lines, $\sigma_i^2 = \sigma_j^2 = 0.45$ corresponding to moderate volatility. In the lines with upward–pointing triangles, $\sigma_i^2 = \sigma_j^2 = 0.80$ corresponding to high volatility. In the lines marked with plus signs, $\sigma_i^2 = \sigma_j^2 = 1.05$ corresponding to very high volatility. The following values of other parameters were used to generate the graphs: the length of the resource lifecycle, $T = 10$; the coefficient of absolute risk aversion, $\gamma = 0.5$; the discreteness with which resource capacity may be redeployed, $L = 10$; the offsets capturing the initial returns, $A_i = A_j = 0.50$; the trends for returns, $\mu_i = \mu_j = 0.1$; the correlation of the innovation terms capturing return correlation, $\rho = 0$; the number of simulated paths for the returns, $n = 10,000$; the risk–free interest rate, $r = 0.1$; and the value of the invested resources, $V_0 = 1$. 

Figure 3. Implications of redeployment costs and return volatility for the dynamics of corporate diversification
Figure 4. Implications of redeployment costs and return correlation for the dynamics of corporate diversification

A.

Figure 4 shows the probability $p_t$ of the firm being diversified in markets $i$ and $j$ in the middle of the resources lifecycle. The first (inverse) proxy for relatedness, the marginal redeployment cost $S$, represents inter–temporal economies of scope and varies within the interval $S \in [0,0.13]$. In particular, $S = 0.0$ corresponds to the strongest possible relatedness between $i$ and $j$ in non–scale free resources, whereas $S = 1.3$ corresponds to very weak relatedness. In both panels, the second proxy for relatedness, the sharing factor $\beta$, is set to the medium value $\beta = 1.00$ with which knowledge relatedness is mediocre and intra–temporal diseconomies or economies are absent. In both panels, the correlation $\rho$ of the innovation terms capturing return correlation takes five values. In particular, in the broken lines, $\rho = -0.99$ representing strong negative correlation. In the lines with downward–pointing triangles, $\rho = -0.50$ revealing weak negative correlation. In the solid lines, $\rho = 0.00$ representing zero correlation. In the lines with upward–pointing triangles, $\rho = 0.50$ showing weak positive correlation. In the lines with plus signs, $\rho = 0.99$ capturing strong positive correlation. The following values of other parameters were used to generate the graphs: the length of the resource lifecycle, $T = 10$; the coefficient of absolute risk aversion, $\gamma = 0.5$; the discreteness with which resource capacity may be redeployed, $L = 10$; the offsets capturing the initial returns, $A_i = A_j = 0.50$; the trends for returns, $\mu_i = \mu_j = 0.1$; the variances of the innovation terms capturing the volatilities of returns, $\sigma_i^2 = \sigma_j^2 = 0.45$; the number of simulated paths for the returns, $n = 10,000$; the risk–free interest rate, $r = 0.1$; and the value of the invested resources, $V_0 = 1$. 

\[ S \in [0,0.13] \]
Panel A of Figure 5 shows the probability $p_T$ of the firm being diversified in markets $i$ and $j$ in the end of the resources lifecycle. In Panel A, the first (inverse) proxy for relatedness, the marginal redeployment cost $S$, represents inter–temporal economies of scope and varies within the interval $S \in [0.0, 1.3]$. In particular, $S = 0.0$ corresponds to the strongest possible relatedness between $i$ and $j$ in non–scale free resources, whereas $S = 1.3$ corresponds to very weak relatedness. In Panel A, the second proxy for relatedness, the sharing factor $\beta$, is set to the medium value $\beta = 1.00$ with which knowledge relatedness is mediocre and intra–temporal diseconomies or economies are absent. Panel B of Figure 5 shows the evolution of the probability $p_t$ of the firm being diversified in markets $i$ and $j$ over the resources lifecycle. In Panel B, the marginal redeployment cost takes an intermediate value $S = 0.195$ representing moderate relatedness of non–scale free resources involved in inter–temporal economies. The sharing factor $\beta$ in Panel B takes three values. In particular, with $\beta = 0.90$ (depicted with the broken line), $i$ and $j$ are very weakly related in knowledge, resulting in strong intra–temporal diseconomies of scope. With $\beta = 1.00$ (depicted with the solid line), knowledge relatedness is average, generating neither diseconomies nor economies. With $\beta = 1.10$ (depicted with the line with plus signs), $i$ and $j$ are very strongly related in knowledge, resulting in strong economies. In both panels, the following values of other parameters were used: the length of the resource lifecycle, $T = 10$; the coefficient of absolute risk aversion, $\gamma = 0.5$; the discreteness with which resource capacity may be redeployed, $L = 10$; the offsets capturing the initial returns, $A_i = A_j = 0.50$; the trends for returns, $\mu_{ii} = \mu_{jj} = 0.1$; the variances of the innovation terms capturing the volatilities of returns, $\sigma_i^2 = \sigma_j^2 = 0.45$; the correlation of the innovation terms capturing return correlation, $\rho = 0$; the number of simulated paths for the returns, $n = 10,000$; the risk–free interest rate, $r = 0.1$; and the value of invested resources, $V_0 = 1$. 
Table 1. Previous and novel insights into the implications of relatedness for the dynamics of corporate diversification

<table>
<thead>
<tr>
<th>Questions on dynamics of corporate diversification</th>
<th>Limiting approaches in extant research</th>
<th>Specific implication in previous research</th>
<th>Novel results</th>
</tr>
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<tbody>
<tr>
<td>How does relatedness affect the propensity of a firm to diversify?</td>
<td>Limited recognition of different types of economies of scope</td>
<td>Relatedness monotonously enhances the probability of corporate diversification.</td>
<td>The monotonous positive effect of relatedness on the probability of corporate diversification does not hold with inter–temporal economies of scope, wherein the probability of diversification has an inverted U–shape relationship with relatedness.</td>
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<tr>
<td>Does the restrictive focus on relatedness suffice to predict the proclivity of a firm to diversify?</td>
<td>Limited attention to determinants of economies of scope other than relatedness</td>
<td>The effect of relatedness on the propensity of a firm to diversify is unconditional (that effect does not depend on other determinants of economies of scope).</td>
<td>The effect of relatedness on the propensity of a firm to diversify strongly interacts with inducements.</td>
</tr>
<tr>
<td>How does the propensity of a firm to diversify evolve over time?</td>
<td>Verbal theorizing in the complex context with inter–temporal linkages between corporate diversification decisions</td>
<td>In the long run, the probability of diversification is affected by relatedness linked to intra–temporal economies of scope but unaffected by relatedness linked to inter–temporal economies of scope.</td>
<td>In the long run, the probability of diversification is very sensitive to relatedness present in inter–temporal economies. Moreover, with moderate levels of relatedness involved in inter–temporal economies of scope, the probability of diversification due to such economies remains very high by the end of the resource lifecycle.</td>
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Table 2. Value dynamics over resources lifecycle

| Time t | \( t=0 \) | \( t=1 \) | \( t=2 \) | \( t=3 \) | \( t=4 \) | \( t=5 \) | \( t=6 \) | \( t=7 \) | \( t=8 \) | \( t=9 \) | \( t=T=10 \) \\
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<tbody>
<tr>
<td>Value ( V_{eT} ) accumulated by time ( t )</td>
<td>( V_{e0}^{1} )</td>
<td>( V_{e0}^{1} \times V_{i:1} )</td>
<td>( V_{e0}^{1} \times V_{i:2} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{1} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{2} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{3} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{4} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{5} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{6} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{7} )</td>
</tr>
<tr>
<td>Value ( V_{eT} ) as seen from time ( t )</td>
<td>( V_{e0}^{1} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{1} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{2} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{3} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{4} )</td>
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<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{10} )</td>
</tr>
<tr>
<td>Value ( V_{eT} ) relevant for maximization of ( V_{eT} ) at time ( t )</td>
<td>( V_{e0}^{1} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{1} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{2} )</td>
<td>( {1 \times (1 + r) } V_{e0}^{1} \times F_{3} )</td>
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