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The evolution of the business cycles distribution of countries

Marco Duenas

LEM - Laboratory of Economics and Management
m.duenasesterling@sssup.it

Giulio Bottazzi

bottazzi@sssup.it

Abstract

With this paper we contribute to the empirical analysis of the distributions of the international GDP business cycles and its evolution. In order to measure volatility two types of proxies per country time series were used, the standard logarithmic definition of growth and the denominated cycles coming from filtering with the Hodrick-Prescott Filter. Under both types, we find out the known stylized facts of growth processes in economics, the tend-shape probability distribution and the inverse power law relation between volatility and the GDP size. The dynamic analysis shows that the tails of probability density functions evolve getting fatter together with a decrease of their variance, suggesting an increasing non null probability of finding high amplitude fluctuations. Also, it is observed that the densities were at the beginning asymmetric with relative higher probability of getting positive fluctuations and that the economic system evolves to a more symmetric scenario giving space to the probability of drawing in the negative part. In addition, it is found that the mentioned power law relation has not remained exactly constant. This fact enriches the discussion about the meaning of scaling relations in economic systems.

Jelcodes:E10,C31

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Abstract

With this paper we contribute to the empirical analysis of the distributions of the international GDP business cycles and its evolution. We built a balanced panel of 111 countries for the 1960-2007 period using as unit of analysis the gross domestic product reported in the Penn World Table 6.3. In order to measure volatility two types of proxies per country time series were used, the standard logarithmic definition of growth and the denominated cycles coming from filtering with the Hodrick-Prescott Filter. Under both types, we find out the known stylized facts of growth processes in economics, the tend-shape probability distribution and the inverse power law relation between volatility and the GDP size. The dynamic analysis shows that the tails of probability density functions evolve getting fatter together with a decrease of their variance, suggesting an increasing non null probability of finding high amplitude fluctuations. Also, it is observed that the densities were at the beginning asymmetric with relative higher probability of getting positive fluctuations and that the economic system evolves to a more symmetric scenario giving space to the probability of drawing in the negative part. In addition, it is found that the mentioned power law relation has not remained exactly constant. This fact enriches the discussion about the meaning of scaling relations in economic systems.

JEL codes: E10, C31, R11

Keywords: Economic Development, Growth Distribution, Business Cycles, Scaling Laws in Economics.

1 Introduction

Economic fluctuations along the history have not followed a regular or cyclical pattern not in the short or long term. The comprehension of the sources, persistence and effects of fluctuations, have been a challenge for the macroeconomic theory. Actually, the spectrum of the fluctuations across filtering techniques also have different economic interpretation of the short-long term, (Canova, 1998). But, are there some invariant properties for different sets of cycles? And, are macroeconomic theories able to account for them?

In the long term, neoclassical models are mainly based on convergence to the steady state. Thus, in each economic system the growth rate declines as they approach their own steady state, which is defined by the preferences and technologies available in each country. Thus, in principle, similar countries would converge to the same growth rate. In this way, the theory explains that given that developing economies are farther from the steady state, they grow faster than the developed ones, which lead to catching up. The fixed point, in any theoretical model, is that in the long term growth must be closely related with structural changes of the economies. Thus, in order to have little differences in the growth rates, developing economies would adopt similar technologies and consumer preferences to the advanced economies. For this reason, it is expected that the cross section variance of growth rates would decrease with time, meaning this less volatility for caching up countries. Structural change also implies that countries will have better institutions to bear with uncertainty.

On the short term, the exogenous growth theory postulates that the economies are approaching the steady state while the cycles, most of the time simulated using random shocks realizations on the supply side, send them away of the convergence point. Hence, the shock produces fluctuations that propagate across the economy. These models are not able to explain the persistent nature of the fluctuations neither the changes in frequency and amplitude. Nevertheless, not much attention is put on the effects on volatility in the growth path for economies with different levels of development. After a crisis, the return to high growth in developing economies requires probably bigger efforts than for developed countries which have institutions and infrastructures to stabilize.

Thus, in the long run would be expected country convergence in mean and variance of the growth rates. Given that in these theoretical models it is assumed independence in the fluctuations of firms and households, the aggregate effect of the economy would lead to a normal distribution by the law of large numbers and the central limit theorem in probability theory. However, the empirical evidence in the cross section indicates that the probability density function of growth rates is tend-shaped with a clearly identified heteroskedasticity, expressed as an inverse power law relation between the standard deviation of the growth rates and the country GDP size, (Lee et al., 1998; Castaldi and Dosi, 2009).

The evidence that the growth rates are non-normal distributed in the cross section not only has an important implication on how the macroeconomic theory understands the growth process, but also in the way that it should be modeled in policy exercises. For instance, in Real Business Cycle models, it is assumed that growth shocks are distributed normally. Wrong econometric assumptions could cause biases in the analysis of coefficients and variances, therefore, the tend-shape fact must be taken into account in order to gather reality. The main characteristic of that type of distributions are the denominated fat-tails. Actually, this property could be considered as a robust stylized fact not only in the cross section, but also it appears when a single country time series is used and even under different filtering techniques (Fagiolo et al., 2008) and (Fagiolo et al., 2009).

On the side of the variance, and more precisely the volatility measured as the standard deviation of the growth rates, before the latest crisis, world economy was living an apparent calm period, proved by the remarkable decline of the volatility. Important scholars denominated that period as the *Great Moderation*. There were different types of explanations of this effect. For instance, it was assumed that structural change has endowed economies to absorb shocks or better financial systems and policy makers were playing an important role. Otherwise, if there were no structural change then low volatility must be because shocks were smaller and less frequent, therefore, the same authors are claiming that there was good luck. The heteroskedasticity means that the volatility is bigger for economies with low income, (Fiaschi and Lavezzi, 2003). This volatility has not been constant in the cross section, neither just declining on time. Indeed, it can be observed empirically that periods with high average growth have also high volatility.

Note that growth rates are used as a proxy to measure the volatility of the economic fluctuations. However the output from other specific treatments on the times series are also available, as for instance, Hodrick-Prescott and bandpass filters. This is a sensitive step in the methodology to study the denominated volatility. Noticeably, each filter removes a specific range of frequencies, generating series of 'cycles' with different amplitudes, frequencies and properties in the short and long term. Then, if the volatility of the economic fluctuations is measured as the standard deviation of the extracted 'cycles' of the GDP path, is that heteroskedasticity a robust feature over those proxies? Actually, it has been shown that the business cycles stylized facts are sensible to the filter type, (Canova, 1998). Is that filtered information sensitive to country sizes? Possibly the answer is that no filter is able to remove just endogenous fluctuations in such a way that the distribution in the cross section is homoskedastic, or simply endogeneity and exogeneity are completely entangled.

Given that different filters produce different type of cycles, they offer different characterizations of the short, medium, and long terms. The filter selection defines the reference system of the economic cycles. In particular, the first difference filter, or growth rate, contains the equilibrium fluctuations in the short term. It is quite sensible, providing erratic changes and short correlation. On the other way, the Hodrick-Prescott filter offers a consistent picture for the medium term cycles.[1] In this work we will be focused on the study of the short and medium term, for this reason only these two filters will be used. The long term cycles can be characterized by the remotion of the long term equilibrium path, the first approximation to the study of the probability distribution of the cycles in the long term was presented in (Lee et al., 1998).

From the empirical point of view, it has been found that countries and firms share some similarities in the growth process, (Lee et al., 1998). Precisely, this similarities regard the invariance in shape of the distribution together with the negative scale law relation between the standard deviation of growth and size. This acquaintance has called the attention about the understanding of how firms and countries are organized inside, and in what extend this organization can be explained by the interaction with other firms/countries and/or exogenous factors. These features give further insights for the identification of invariant properties at the level of countries or firms. Although in some cases it is difficult to determine the sources and effects of economic disturbances, the understanding of the specificities could disclose the generating mechanisms underlying economic growth and the processes of diffusion of technological and demand shocks (Castaldi and Dosi, 2009).

The scaling laws are related with the property of having straight lines independently of the period of observation (Brock, 1999). In this view, the scaling laws become properties of stationary distributions and,

[1] Most of the literature of Real Business Cycle use the HP-filter to remove the trend the time series.

hence, cannot say much about the dynamics of the stochastic process which generated them. Nevertheless, it should be pointed out that the existence of that kind of properties reduces in some way the set of possible stochastic processes. Many studies of the GDP growth distribution after the Second World War have stated that the distribution was stationary and that the scale relation was linear with slope $\beta = -0.15$ and has remained constant during this period. However, the methodology proposed here shows that actually this scale has changed in this period.

Those features have opened questions about the type of stochastic processes behind the growth process. The understanding of the evolution of the probability distribution can allow us to establish if economic fluctuations require more attention or if other factors can modulate the frequency and amplitude, for instance economic crisis or spread of institutional policy frameworks. But also the evolution of the heteroskedastic term that accounts for the internal interdependences of the countries could give us some intuition about the effects that fluctuations have on the country economic structure. Assuming that structural change must be closely related to volatility, we analyze the effects of the middle term given by the HP-cycles and the short-term given by the growth rates.

In the section 2 the details of the datasets and the definitions of the variables are presented. In all the paper are shown in parallel the estimations for the growth rates and the cycles of the HP-Filter in order to distinguish differences. Thus, we derive the parameters of the probability density function for the pooled sample in the section 3 together with the characterization of the power law relation between volatility and country size. In section 3 our approach to the problem is introduced, allowing the comparison with previous results shown in the literature. The dynamic generalization of the model is presented in the section 4, all estimations are presented in figures with the corresponding confidence intervals. The section 5 discusses the estimations in three time intervals. The section 6 presents the main conclusions.

2 Data and definitions

For the cross section analysis, it is used the Penn World Table 6.3 which is already reduced in real terms using as base the 2005 year. More precisely the constant price GDP for a balanced panel starting from 1960 until 2007 with 111 countries is used. The variable $y_{i,t}$ denotes the natural logarithm of the GDP for the country i in the year t.

We will use two different units of analysis in order to derive the GDP volatility. On the one side, the cycles after applying the Hodrick-Prescott Filter (HP-filter) on each log-GDP country time series, which are characterized by high frequency fluctuations and stationarity. The HP-filter captures the non-linear long term path for the GDP filtering low frequency fluctuations. In general, the HP-filter setup assumes that log-GDP time series can be decomposed by a trend component θ plus a cyclical component c. Thus, the cycles can be written as

$$c_{i,t} = y_{i,t} - \theta_{i,t}. \tag{1}$$

Technically, those cycles can be calculated using the standard minimization problem proposed by Hodrick-Prescott with the parameter $\bar{\lambda} = 6$, given that we are using annual data. We will make reference to the set of cycles $c = \{c_{i,t}\}$ as the HP-cycles.

On the other side, we used the first difference operator on the log-GDP (r=Δy), the standard log-growth rate, which is characterized by leading pass a wider band of frequencies and non-stationarity. Then, the set of growth rates $r = \{r_{i,t}\}$ are defined as

$$r_{i,t} = y_{i,t} - y_{i,t-1}. \tag{2}$$

From the statistical point of view there are many differences between HP-cycles and growth rates. The main ones are that in the former the fluctuations are greater and contain the low frequency movements generated by the country specific economic cycle. Both definitions are sensitive to exogenous factors, but certainly the HP-cycles have less endogenous information and idiosyncratic perturbations.

Size measure. Given that different countries have different levels of GDP is was needed to define a relative measure of country sizes in order to be able to compare them in the cross section analysis. This size measure is defined as

$$S_{i,t} = y_{i,t} - \overline{y_t}. \tag{3}$$

Note that the average of $S_{i,t}$ in the cross section is zero, leading countries with low GPD to have negative sizes and the opposite in the case of high income economies. One of the goodness of this measure is that, along the row, the trend is removed eliminating common components to all countries.

Probability Density Function (PDF). The known tend-shape issue of the growth rates distribution in the cross section implies that if it is compared with the reference Normal Distribution, it can be observed that the tails are fatter, falling in an exponential way. Indeed, some theoretical approximations suggest that the density function is Laplacian, (Lee et al., 1998; Castaldi and Dosi, 2009). Hence, one useful framework to study the empirical probability densities of our set of variables, $\{r, c\}$, is to fit them to the family of Subbotin densities, methodology that was introduced by Bottazzi and Secchi (2003). In this work, we use the general asymmetric form of the Subbotin density function, which can be written as

$$f(x, a, b, m) = \begin{cases} \frac{1}{A} e^{-\frac{1}{b_l} |\frac{x-m}{a_l}|^{b_l}} & x < m \\ \frac{1}{A} e^{-\frac{1}{b_r} |\frac{x-m}{a_r}|^{b_r}} & x > m \end{cases} \tag{4}$$

where,

$$A = a_l b_l^{1/b_l} \Gamma(1 + 1/b_l) + a_r b_r^{1/b_r} \Gamma(1 + 1/b_r),$$

where $x \in \{r, c\}$, the parameter $a_{\{l,r\}}$ characterizes the variance, $b_{\{l,r\}}$ the shape, for left and right tails, and m the position of the center. Note that the symmetric version is recovered if the parameters from the left and right are equal. For instance, the Normal and Laplace distributions belong to this family for the values $b_{l,r} = 2$ and $b_{l,r} = 1$, respectively. For a detailed description of the estimation, see also (Bottazzi, 2004).

On the side of heteroskedasticity, it is expected a negative power law relation with the country sizes. This relation can be formulated as

$$\ln(\sigma_x) \sim \beta_x S. \tag{5}$$

In the case of growth rates this means that high GDP economies are less volatile than low economies. In the next section we show that this relation is also found for the HP-cycles.

3 Scaling growth and cycles

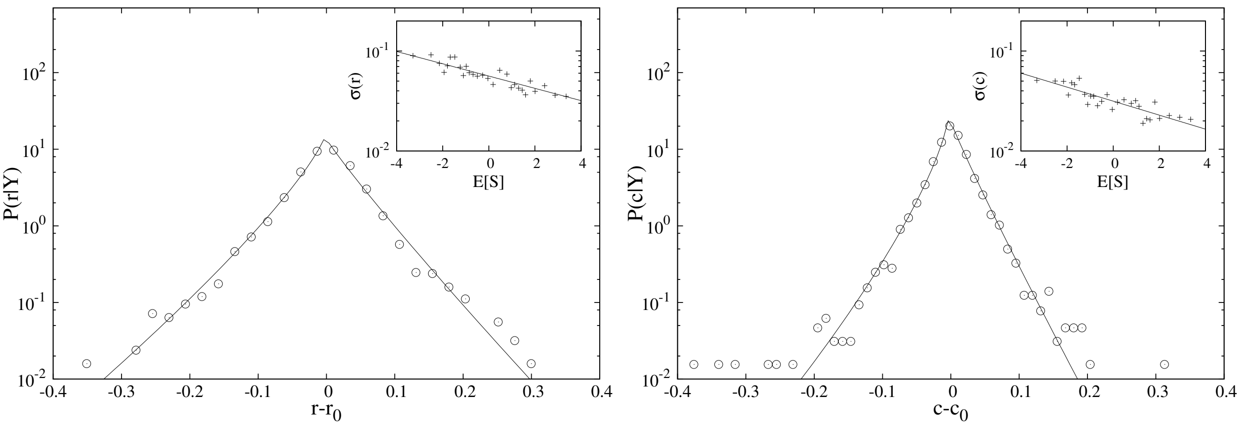

Figure 1: Probability density functions for growth rates (left) and HP-cycles (right), for 1960-2007 pooled data using 60 bins. The points correspond to the binned empirical densities and the continuous lines to the asymmetric Subbotin estimation. In each figure is added a sub-plot with the corresponding binned statistics σ_x vs. S relation, where the straight lines correspond to the OLS estimations using 30 bins.

Our objective in this section is to see if the already mentioned features for growth rates also fit under HP-cycles. Nevertheless, the calculations are done for both in order to recognize differences among them. We start replicating the methodology implemented by (Lee et al., 1998) and Castaldi and Dosi (2009), and letter we formalize the model we will follow in the dynamic part.

Pooling the observations for all years, we have calculated the empirical densities as can be observed in Figure 1. We performed the symmetric and asymmetric estimation for r and c and the results are shown in the Table 1 in the columns denominated Non-scaled PDF. Moreover, inside each figure is added a sub-plot in semi-log scale with the corresponding heteroskedastic relation in order to recognize easily the power law relation shown in the equation (5). The straight line belongs to the OLS estimation of the bin statistics using 30 bins. To be more precise, the standard strategy of using binned statistics on pooled country information is to arrange in one column the sizes and in another one the corresponding growth rates (or HP-cycles), then,

Classes	Non-Scaled PDF			Binned-scaling			MAD-scaling		
	b	b_l	b_r	b	b_l	b_r	b	b_l	b_r
Growth-rates	0.87	0.82	0.92	0.94	0.89	1.00	0.91	0.88	0.95
	(0.02)	(0.03)	(0.03)	(0.02)	(0.03)	(0.04)	(0.02)	(0.03)	(0.03)
HP-cycles	0.83	0.74	0.93	0.93	0.87	1.01	0.92	0.85	1.00
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.02)	(0.03)	(0.04)

* Standard errors are reported in parenthesis

Table 1: Estimated shape parameters of the PDF by Subbotin Distribution for the period $\tau \in (1960, 2007)$, without scaling, scaled by binned estimations equation (6), and by MAD regression equation (8). The b labeled without subscript makes reference to the symmetric Subbotin estimation while b_l and b_r to the asymmetric. For technical details of the estimation of Subboting Densities see (Bottazzi, 2004).

sorting by size, the sample is divided in different bins and in each binned sub-sample is measured the mean of the sizes and the standard deviation of the growth rates (or HP-cycles). Thus, using OLS is possible to determine the slope of the relation between $\ln(\sigma_x)$ and S.

These results highlight the robustness of heavy tails in the growth process. Thus, recapitulating the already known features for the growth rates and our estimations, we can see that: i) the tend-shape is observed for both HP-cycles and growth rates but, for the last one, the domain is greater because it contains higher fluctuations (from the average). Regarding the symmetry of the PDFs, the results of the Table (1) show that statistically growth rates are slightly asymmetric, whilst HP-cycles are not; ii) under the 'size' definition the presence of heteroskedasty is confirmed, and, more precisely, in both cases it is adjusted to a power law relation. For growth rates the calculated slope is $\beta_r = -0.142(0.012)$ and for HP-shocks we get $\beta_c = -0.161(0.014)$.

Rescaled variables. The consequence of the identified heteroskedasticity is that in the equation (4), the parameter a that characterizes the variance in the distribution can no longer be considered constant and must be generalized $a \to a(x|S)$, but given that it was possible to estimate β_x, a re-scaling of the variables would lead to a homoskedastik distribution. In the case of the growth rates, Lee et al. (1998) proposed to rescale by the term $\exp(\beta_r S)$. Thus, in general we can rescale our set of variables as

$$x'_{i,t} = \frac{x_{i,t} - x_0}{e^{\beta_x S_{i,t}}}, \tag{6}$$

where, $x \in \{r, c\}$ and x_0 is the average of the whole sample. Note that our size measure is used for scaling the growth rates and HP-cycles.

In the Table 1 is shown the Subbotin estimation of the rescaled distributions. In the symmetric estimation, the shape of the tails for both rescaled distributions r' and c' are quite similar. Nevertheless, in the asymmetric case, the fact that for the rescaled HP-cycles the distance between b_l and b_r is greater than the rescaled growth rates suggests more asymmetry for c'. The fact that the right tail can be considered statistically, means that the growth process is described by the Laplace distribution.

Note that in the Table 1 are not reported the estimations for the variance parameter $a_{l,r}$ because under two significant digits they where the same, even without rescaling. The values for growth rates were $a = a_l = a_r = 0.04$ and for HP-cycles $a = a_l = a_r = 0.02$.

3.1 Other approach for rescaling

In the previous section we have seen that the tend-shape is a common feature of the growth rates and HP-cycles. We have also seen that these distributions are almost Laplace in the sense that the estimated b-parameters are almost always slightly less that one. Having this in mind, the purpose of this section is to present a different approach to the problem of identifying and removing the heteroskedasticity of the distributions having into account the shape of the distribution, strategy that has been used by Bottazzi and Secchi (2003) and Castaldi and Dosi (2009).

From an empirical point of view, either the log-GDP growth or the HP-cycles series are significantly autocorrelated. Hence, in general x can be described by the process,

$$x_{i,\tau} = \alpha_{x,\tau} + \lambda_{x,\tau} x_{i,\tau-1} + u_{i,\tau}, \tag{7}$$

Parameters	Growth-rates		HP-cycles	
	Binned-OLS	MAD-estimation	Binned-OLS	MAD-estimation
β	-0.142	-0.135	-0.161	-0.157
	(0.012)	(0.005)	(0.014)	(0.005)
λ		0.255		0.183
		(0.009)		(0.009)
α		0.030		0.000
		(0.001)		(0.000)

* Standard errors are reported in parenthesis

Table 2: Estimated parameters of the stochastic process, for the period $(1960, 2007)$, using OLS on the binned statistics and MAD regression equation (8).

where $\alpha_{x,\tau}$ is the constant term that converges to the average of x in time interval τ, and $\lambda_{x,\tau}$ the autoregressive term. Thus, considering the already observed functional form of the heteroskedasticity, the error term can be written as $u_{i,\tau} = e^{\beta_{x,\tau} S_{i,\tau}} \epsilon_{i,\tau}$, then replacing this expression on the previous equation and clearing on the error term ϵ we have

$$\epsilon_{i,\tau} = \frac{x_{i,\tau} - \lambda_{x,\tau} x_{i,\tau-1} - \alpha_{x,\tau}}{e^{\beta_{x,\tau} S_{i,\tau}}}. \tag{8}$$

This model allow us to measure and correct the heteroskedasticity, in agreement with the equation (5). Note that, in this framework, the distribution of ϵ is homoskedastic and equivalent to the distribution of the rescaled variables, which is our final objective.

There are different strategies to minimize the errors of the non-linear equation (8). In general, it is used a Linear Least Square (LLS) regression, however, we want to highlight the implementation of the Minimum Absolute Deviation (MAD) regression, because agrees with the observed shape of the PDF and, therefore, it is less bias. Under the MAD estimator the problem can be written as

$$\{\beta_{x,\tau}, \lambda_{x,\tau}, \alpha_{x,\tau}\} = \arg\min_{\beta,\lambda,\alpha} \sum_{i,\tau} \left| \frac{x_{i,\tau} - \lambda x_{i,\tau-1} - \alpha}{e^{\beta S_{i,\tau}}} \right|. \tag{9}$$

Note that this expression is proportional to the log-likelihood function when it is a Laplace distribution. The subscript τ is used in the equation (9) to make reference to the interval of years to be pooled.

In the Table 2 are summarized the results of the binned-OLS estimation of the heteroskedastic term and the MAD estimation equation (8) for the whole sample period. First, the expected value for β is always greater for MAD estimation, although statistically equal. Second, MAD estimation is more accurate. Third, the autoregressive term λ is significant as was expected because growth rates are more autocorrelated. And forth, α_r means that in the whole sample the average growth was 3%, while for HP-cycles α is zero, given that by construction the cycles of HP-filter have zero mean.

In the Table 1 are summarized the results of estimations of the PDFs of the estimated residuals equation (8). There are not important differences between the binned strategy and the MAD estimation, HP-cycles are more asymmetric than growth rates.[2] Indeed for the estimation were used 47×111 observations enough to get agreement in both strategies. Nevertheless, note that under the MAD estimation we were able to grasp more features of the stochastic processes without depending on the selection of the number of bins.

4 Dynamic of the parameters

Lee et al. (1998) suggests that after the Second World War the growth rate distributions is well described by the symmetric Laplace Distribution, remaining constant together with the parameter β. Our estimations show that the Laplace fits quite well for the right tails in the whole sample period and even for HP-cycles. Nevertheless, the left tail is always heavier. Our objective in this section is to test if the empirical densities are indeed stationary and asymmetric, taking advantage on the fact that the MAD estimation is more accurate.

The problem with the use of bin statistics is that needs to pool many years and country data of different years are mixed, leading (possibly) to impressions. For instance, if fifty years are pooled, it could be that

[2] Castaldi and Dosi (2009) reported a greater measure for the GDP growth rates using the period 1960-1996. We show that the differences with our results are explained by the empirical fact that in the mid 1990s the tails of the distributions started to get heavier.

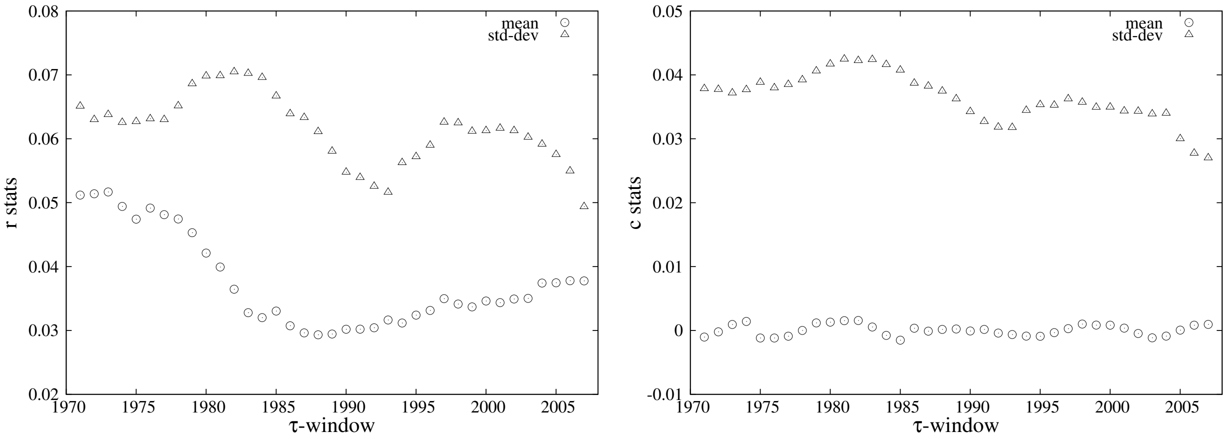

Figure 2: Dynamics of the first two moments of the growth rates (left) and HP-cycles (right) in the moving window

the income of rich countries at the 1960s is compared to the income of some emerging countries in the last years, given that theoy could have comparable sizes but in different periods, and possibly their performances were different. In other words, in fifty years are mixed different countries sizes, world economic fluctuations, economic cycles, different policies, technological paradigms, etc. Actually, those factors can modulate the amplitude of the changes in the GDP depending of the country sizes.

The methodology proposed in the last section can be applied in different consecutive periods τ in order to analyze the evolution of the distributional parameters. The strategy that will be followed here is to use a moving window time-period of ten years, hence in every realization will be available 111×10 observations. The cost of reducing the whole period is that the regressors could be less significant, from the statistical point of view, but may be richer in the economic context.[3] In order to clarify the notation for the time from now on, a time period $\tau = year$ is referred to the time interval $(\tau - 10, \tau)$, in other words, the analysis for a given year correspond to the pooled information 10 years before.

In the Figure 2 is shown the evolution of the two first moments of the growth rates in the cross section, using the moving windows. The evidence suggests that, the pooled mean started to fall rapidly from 5% at the beginning of the 1970s to 3% at the beginning of the 1990s, followed by a period of low rise until the end of the sample arriving to 3.8%. The standard deviation has a tendency to decrease in the long run although in a more irregular way. The same figure shows also the evolution of the two first moments of the HP-cycles. As expected, the mean oscillates around zero with low amplitude and, again, the standard deviation has a long run tendency to decrease. The fact that the variance of both, r and c, exhibit this decreasing tendency could suggest that also changes in β_x may be expected, through equation (5). Actually, there is no theoretical reason to believe that it has remained constant.

In the Figure 3 are plotted the estimation of the parameters $\{\beta_x, \lambda_x, \alpha_x\}$ for the growth rates on the left and HP-cycles on the right panel. From now on, all error bars added to each plotted point corresponds to two times the standard error of the estimation. The β_r estimated follows an increasing-decreasing smooth-like trajectory. Although the movement is not monotonous, it is possible to identify two different regions. At the beginning is constant until the last quarter of the 1980s, around $\beta_r = -0,15$. Then, the rest of the period is observed a complete oscillation that reaches the highest $\beta_r = -0,11$ in 1993 window, the lowest $\beta_r = -0,17$ in 1998, and again at the end of the period around $\beta_r = -0,11$. On the side of the HP-cycles, the behavior is slightly the same without the tendency to grow since the middle of the 1990s. Actually, β_c seems to be flatter (constant) in the long term, however, it is evident that in the transition from the 1980s to the 1990s the dynamic was different. Finally, the estimated β_r is greater than β_c, although the error bars overlap.

Both autoregressive coefficients are significant, and growing in the long term without flat regions. Regarding the constant factor α, their chances are proportional to the mean, as was expected from the equation (8).

Symmetric PDF. Now, we move the analysis to the estimation of the density function in the moving window for the re-scaled variables, which are given as the residuals of our model, see equation (7). The results

[3]We found that for the estimation of the equation (9), pooling all the countries, it is enough to use two consecutive years in order to have good estimations, while in the case of the probability distribution estimations, equation (4), is needed to use more observations. We observe that for around ten years the estimations behave quite well in the whole sample period.

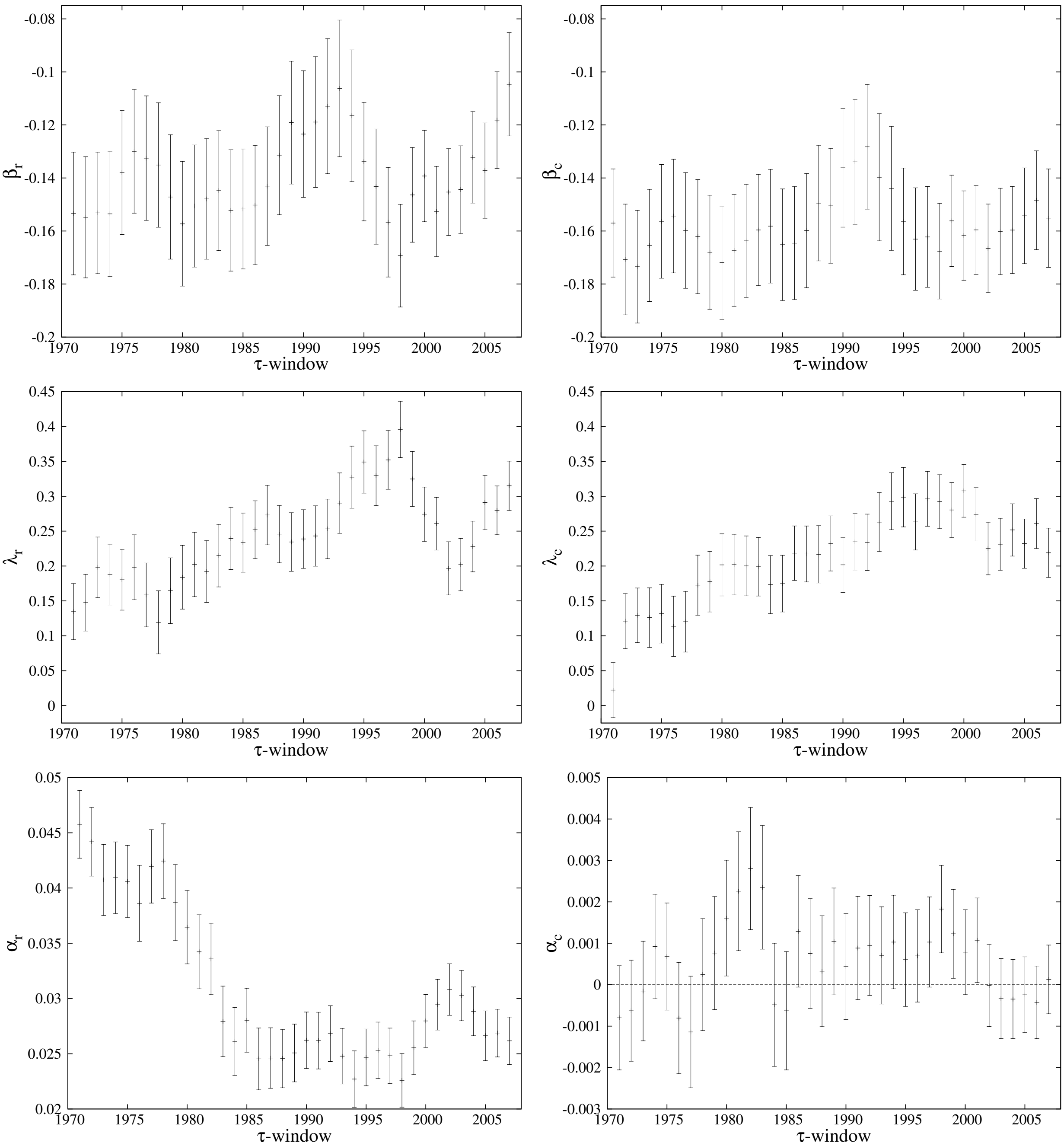

Figure 3: Estimated parameters of the stochastic equation (7). Error bars correspond to two times +/− the standard deviation of the estimated parameter. Left panel growth rates and right panel HP-cycles. Data pooled in a moving window of ten years.

are shown in the Figure 4 with error bar.

The results are quite similar under both units of analysis. The trajectory followed by the parameter b is decreasing in the long run. It means that the tails are getting fatter. Nevertheless, two different regions are identify, almost until the middle of the 1990s the level is statistically constant taking a value just greater than $b = 1$, afterwards it starts decreasing until the last observed year. Considering the error bars, it can be concluded that the Laplace distribution can account for an appropriated description of the economic shocks, however, in the last years the confidence interval for the parameter b takes vales between 0.7 and 0.9 encouraging the issue of fat tails.

The parameter a decreases in the long run. Two different regions are identified, in the first one, it increases monotonously until the mid 1880s, then it starts decreasing slightly more rapidly and monotonously as well

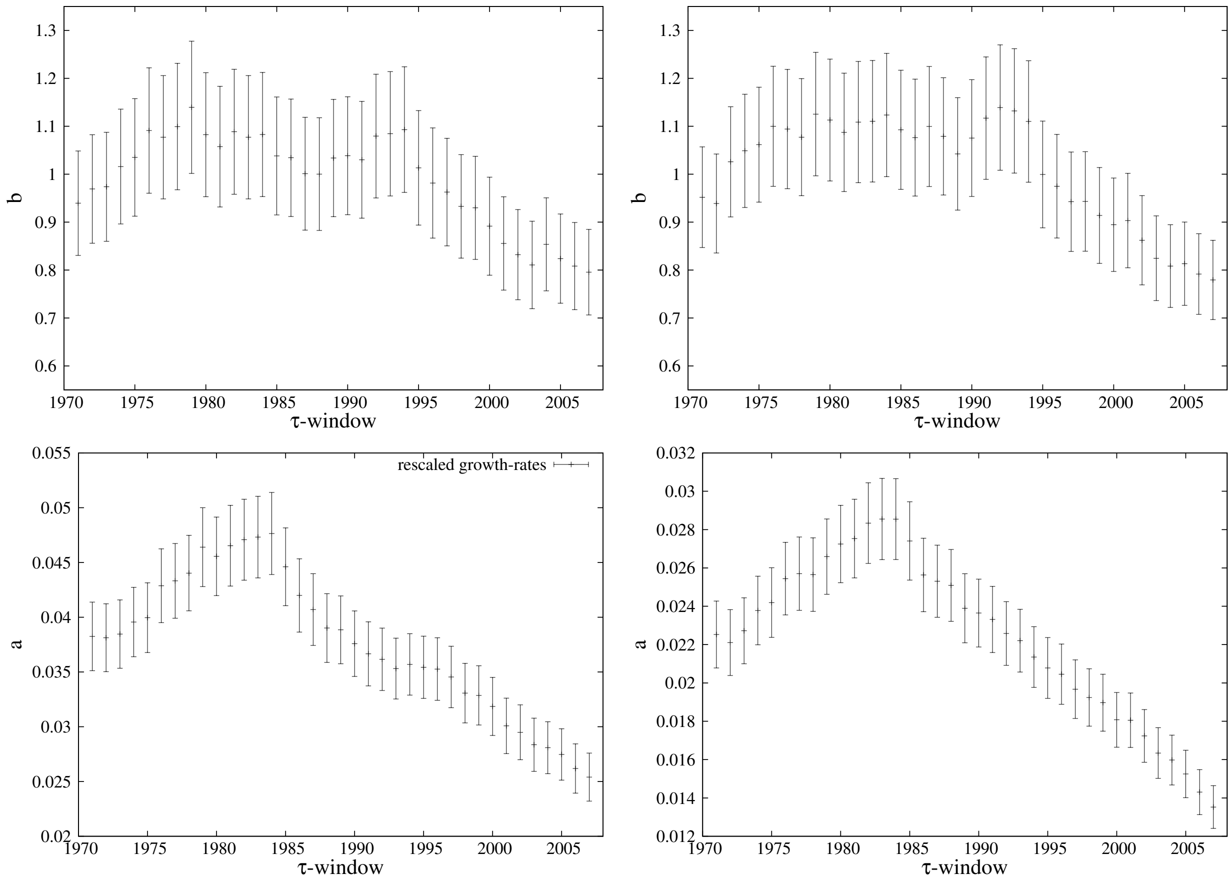

Figure 4: Estimation of the PDF of the residuals of the stochastic equation (7) using the symmetric Subbotin family. Error bars correspond to two times +/− the standard deviation of the estimated parameter. Left panel growth rates and right panel HP-cycles. Data pooled in a moving window of ten years.

until the last period of observation. As expected, this parameter is greater for growth rate.

These estimations show almost no difference in the evolution of both density functions. The facts that at the same time the PDF is getting fatter ($0 < b < 1$) and the dispersion parameter shrinks ($a \to 0$) mean that even if our units are getting less volatile it is also observed a non null probability of having high amplitude fluctuations. Lower a represents that many (but not most) of the fluctuations fall in the center of the 'tend-shape', where the probability intensity is higher. These effects could be interpreted as a weak type of convergence, in the sense that the probability of drawing shocks near to the mean is getting higher. However, to speak about convergence under non stationary parameter and fat tails could be difficult.

Nonetheless, the growth rates and HP-cycles residuals might be differentiated in their behavior under and below the mean. In other words, considering the estimations of the whole sample, we still need to see if they are and/or have been symmetric.

Asymmetric PDF. In the Figure 5 are shown the estimation for the parameter b and a left and right.

In the case of the growth rates, we observed that the right tail is characterized by decreasing a and b, which can be understood as probability losses of having positive shocks in the course of the years. Actually, starting the moving window of the 1970s, the parameters that describe the left tail were much lower than the right tail ones, pointing that low growth was less probable. Nevertheless, the left tail pumped rapidly reaching the right tail in the following years. Note that while the parameters of the right tail fall monotonously the route of the left-parameters is more like an inverse U.

For the HP-cycles the dynamics is different. The first clear feature is that the estimations oscillate displaying higher amplitudes, and seemingly lower frequency. Notwithstanding the high fluctuations, the right tail gets fatter in the long run, while the tendency of the left tail is more stationary; although it calls the attention the sudden high increase at the end of the 1980s.

These results show that the probability density functions were not symmetric in the period of study.

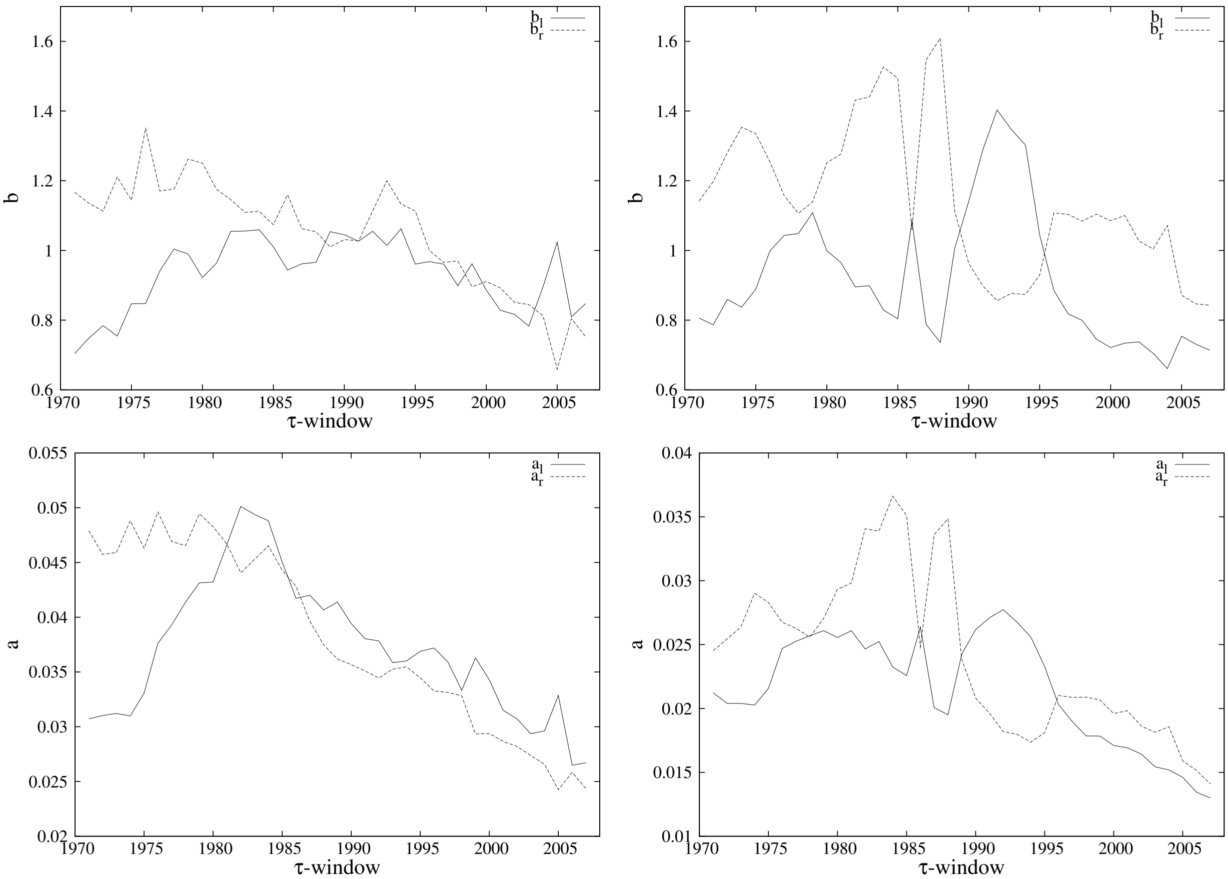

Figure 5: Estimation of the Asymmetric Parameters of the Subbotin distribution. Error bars are omitted for visual reasons. Left panel growth rates and right panel HP-cycles. Data pooled in a moving window of ten years.

Nonetheless, it seems that there was a process of symmetrization. In order to see this, we have plotted the distances between the right and left parameters, see Figure 6. Note that statistically the growth rates started to be symmetric more or less since the moving window of the 1980s, both in shape and variance, in b and a, correspondingly.

For the HP-cycles, it is difficult to accept the symmetry. Note that there are just two moments where the left tail was higher, the less remarkable at the end of the 1970s, and a huge sinking in the transition from the 1980s to the 1990s. The fact that there are no stable periods around the zero line is a good indicator of the asymmetry of the HP-cycles.

Note that the increase in the variance observed in the symmetric estimation, is explained by variation of different tails of the distribution of the asymmetric estimation. To be more precise, for instance, until the mid 1980s, the increment in the symmetric estimation for the growth rates is explained by the increment on the variance of the left tail, while for the HP-cycles is given by the variation in the variance of both tails.

5 Discussion

In order to better understand what happen with all the parameters at the same time, let us split the time-windows. Actually, the estimated parameters do not change synchronously, nevertheless it is possible to identify some intervals with almost unchanging dynamic patterns for the whole set of parameters. Based on our results, we propose to analyze the whole dynamics in three intervals of moving windows: from the beginning until mid 1980s, then until the mid 1990s, and later until 2007.

1970s to mid 1980s windows. As we have already seen, this period is characterized by the most dramatic fall in the average of growth rates, although its standard deviation remained almost unchanged, matching with

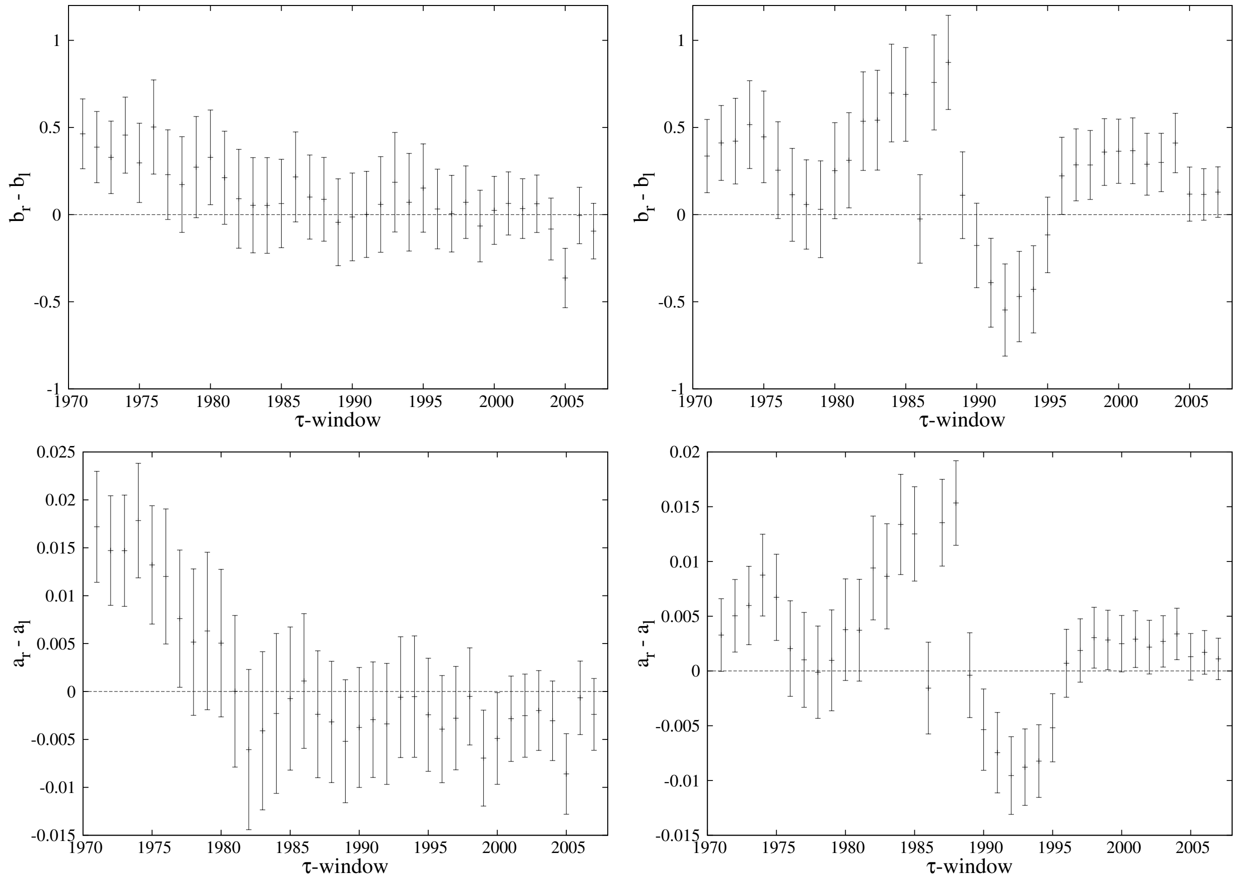

Figure 6: Distance between the parameters for the right and left estimations. Error bars correspond to two times +/− the standard deviation. Left panel growth rates and right panel HP-cycles. Data pooled in a moving window of ten years.

the flatness of β_r. Moreover, the autoregressive term started increasing highlighting that the system became more persistent, meaning that nearest pass weights more.

The interesting phenomenon of this period is the symmetrization of the tails of the growth rates density function. The probability losses of having growth over the average lead to the reshaping of the tails. The right tail gets fatter meanwhile the left gets thinner (less fat), indeed around the middle of the 1980s $b_l = b_r = 1$. In the symmetric scenario, the distributional measure of the variance (a-parameter) increases window per window until the maximum. The asymmetric estimation allowed us to see that this increase is driven by the variance of the left tail, in agreement with the increasing behavior of b_l.[4]

Thus, the probability losses of having growth over the average became in growth under the average, but not at the left extreme. The fact that b_l arrives to 1 is a symptom of extinction of extreme low (even negative) growth. Nevertheless, this combined with low average growth can be translated in low economic performance.

There is not much to say about the HP-cycles in this period as the dynamic is quite similar. However, the positive asymmetry of the density function is there, and changes in left-right oscillate more with higher amplitudes than the growth rates; high b_r and not very low b_l are observed. If we believe that they are composed mainly by exogenous factors, then this was a positive period for the growth of the economies, contrasting with the fall of the average growth.

Middle 1980s to middle 1990s windows. During this period, the growth rate registered the lowest average, around 3%, the standard deviation decrease from 7% to 5%. The autoregressive term continues increasing.

This is the period where growth rates are better explained by the symmetric Laplace distribution. As

[4]Imagine a tend perfectly close with no air leak with you inside. Now, at the beginning you push-up the right side so now there is less air on the left part (the fat tail $b_l < 1$). The distance from the center to the right side is greater than to the left ($a_r > a_l$). Then stop pushing gradually, arriving to $b_l = b_r$.

expected from the falling in the variance, also the distributional variance starts decreasing. This means that in the center the density function, the perfect tend is getting higher by reducing the distance between the tails. This is like a period of stabilization, because negative fluctuations are each time window less probable, which contrasts with the low performance.

What is a puzzle is the change of the scaling parameters β_r and β_c. The increase of this power means that there is less heteroskedasticity. Countries with slightly similar sizes face equivalent volatilities. If $\beta \to 0$, then the volatility faced in the growth process would be independent of the sizes. Actually, the evolution of this parameter in the whole sample is inversely correlated with the sample standard deviation. Thus, statistically, for low world wide volatility less negative (flat) the heteroskedastic term.

The dynamic of the HP-cycles are somewhat contrasting with the growth rates. The PDF is negative asymmetric, in some windows $b_l > 1 > b_r$. Just for this period, high amplitude negative cycles, or 'exogenous' negative fluctuations, are less probable than positive. Nevertheless, this also means more probability for negative cycles near to the center.

Middle 1990s to the end window. This is a period of slow increments in the average growth rate. However, parameters like the sample standard deviation, the autoregressive term and the power relation make half oscillation.

For both probability distributions is observed the fattening of the tails. With the same tendency, the distributional variances continue decreasing as consequence of the tails behavior. This configuration provides increasing probability of having extreme shocks. In the case of the growth rates, the left tail behaves in the same way as in the 1970s windows. Finally, the right tail is continuously deflated.

6 Conclusions

This paper has studied the cross section evolution of the probability distribution of the GDP growth rate and the cycles extracted by Hodrick-Prescott Filter to the GDP time series. We have considered these set of variables as two different proxies of the economic fluctuations. The implementation of a non-linear dynamic model allowed us to identify the heteroskedasticity and to re-scale the fluctuations. The conclusion of the estimations can be divided in two parts, one related to the parameters of the non-linear model and the other about the analysis of the distribution of the rescaled business cycles.

Endogeneity is observed at different frequency levels of the business cycles. We have proved that also HP-cycles are heteroskedastic, leading to a power law relation among the size and the volatility. The functional form is quite similar to the already known for the growth rates. Actually, pooling the whole period of study, it can be concluded that the estimated exponents for growth rates and HP-cycles are statistically equal. The dynamic estimation reveals that for growth rates, this exponent fluctuates more than HP-cycles. Howeever, in both cases are not observed important changes in the parameter. This feature agrees with the fact that growth rates are more sensitive in the short term. While for the HP-cycles the dynamic flatness of the exponent is related with the fact that in the middle term are not expected important changes in the distribution of the volatility caused by business cycles.

In macroeconomic theory, it is widely accepted that the way in which countries handle volatility is closely related with their structures.The fact that the power law between size and volatility does not change a lot in different periods implies that neither the interdependences in the economic structure do. This point is probably clearer for the HP-cycles, where the power relation change less. Thus, we did not observe important changes in the economic interdependences of the countries during the period of study. Nevertheless, we call the attention that, approximately, after mid 1980s, business cycles have had more important effects on the short term distribution of the country interdependences, as was pointed for growth rates in the figure 3.

Regarding the probability distribution of the rescaled business cycles, we observed that also HP-cycles have fat tails. But, even more important we have shown that in both growth rates and HP-cycles the tails of the density function get fatter monotonously with the pass of the time. This result not only contrast with the statements of the *Great Moderation* literature but also implies that convergence theories should be more aware of the features of business cycles.

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